-Tetronimo Tilings

Markov-Chain mixes fast

Flips and Flip-Graphs 2022

Marcel Boesl

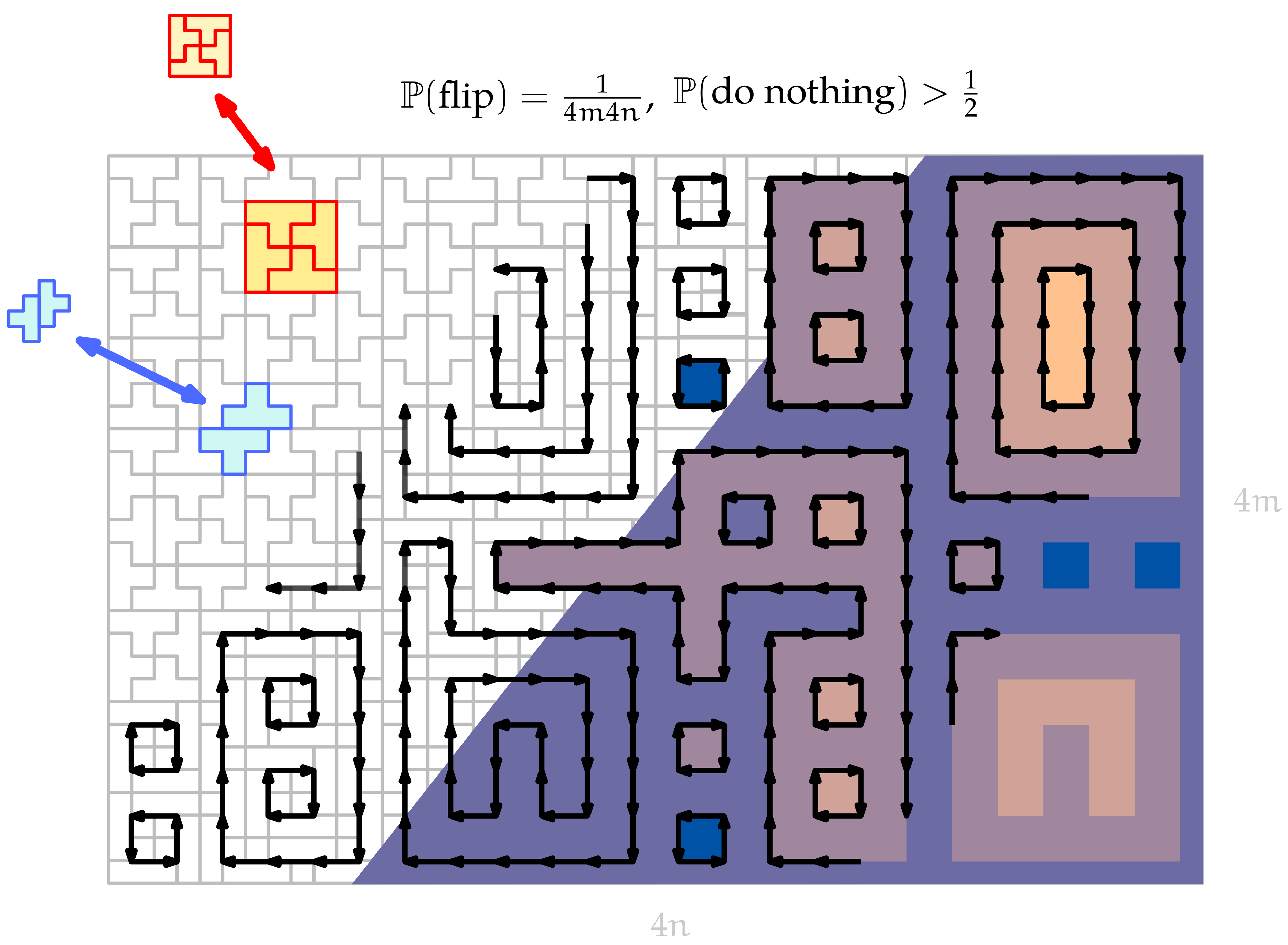
Theorem (Kayibi and Pirzada 2018). *The T-Tetronimoes Markov-Chain mixes fast:*

$$\tau(\epsilon) \leqslant 2(4m)^4(4n^4)\log(\frac{2}{\epsilon})$$

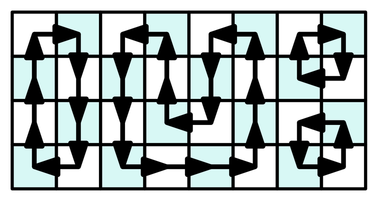
Thus there exist a fully polynomial randomized approximation scheme (FPRAS) for counting T-Tetronimo tilings.

Flip-Graph of T-Tilings:

$\mathbb{P}(\text{flip}) = \frac{1}{4m4n}$, $\mathbb{P}(\text{do nothing}) > \frac{1}{2}$



Useful Bijections for T-Tilings:

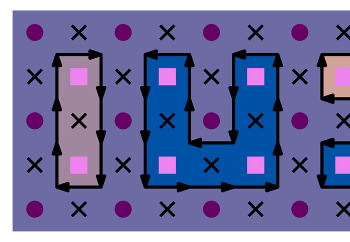


Chain Graph (Walkup 1965)

- $V \leftarrow$ blocks
- Arcs are horizontal or vertical and have length two
- Degrees: $\delta^+ = \delta^- = 1$
- B-type antiblocks border exactly two non-adjacent edges

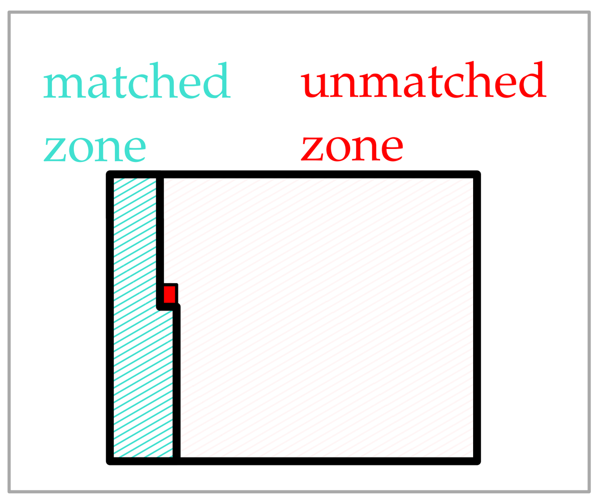
Height Function (Korn and Pak 2003)
f : Antiblocks $\to \mathbb{Z}$ such that

- $f(x) = 0$ on the border
- $f(x)$ is even for $x \in A0$
- $f(x)$ is odd for $x \in A1$
- $|f(x) - f(y)| \leqslant 1$ for x, y adjacent



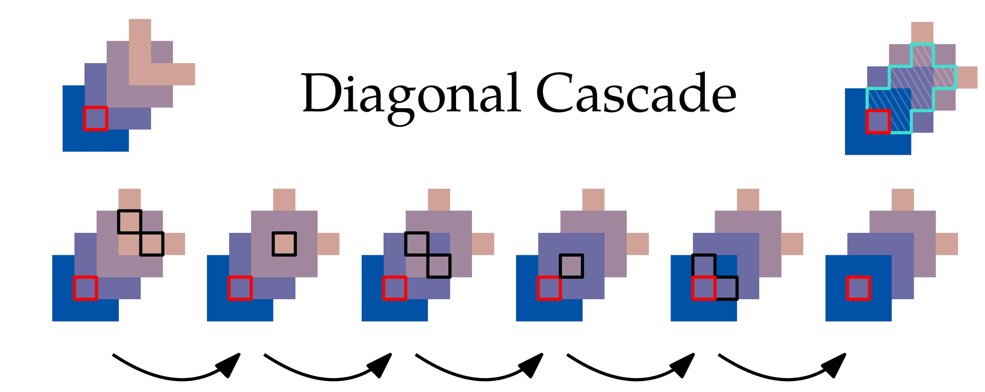
Canonical Paths

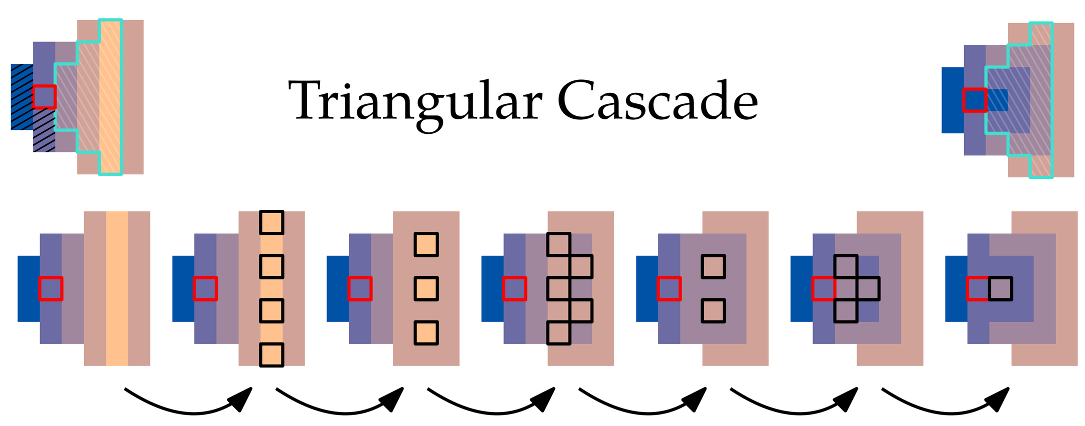
Data: X, Y height functions
Result: CanonicalPath $(X \to Y)$.
if *not all points are matched* **then**
  $u \leftarrow$ smallest unmatched point;
  if u *is not pivotable* **then**
    make x_i pivotable via $S(u)$;
    $X \leftarrow \text{pivot}(X, S(u))$;
  end
  $X \leftarrow \text{pivot}(X, u)$;
  return $S(u)$, u, CanonicalPath $(X \to Y)$
end



Lemma (Kayibi and Pirzada 2018). *The subpath $S(u)$ can stay in he unmatched zone. Thus any canonical path has length*

$$|\mathit{CanonicalPath}(X \to Y)| \leqslant (4n)^2(4m)^2$$



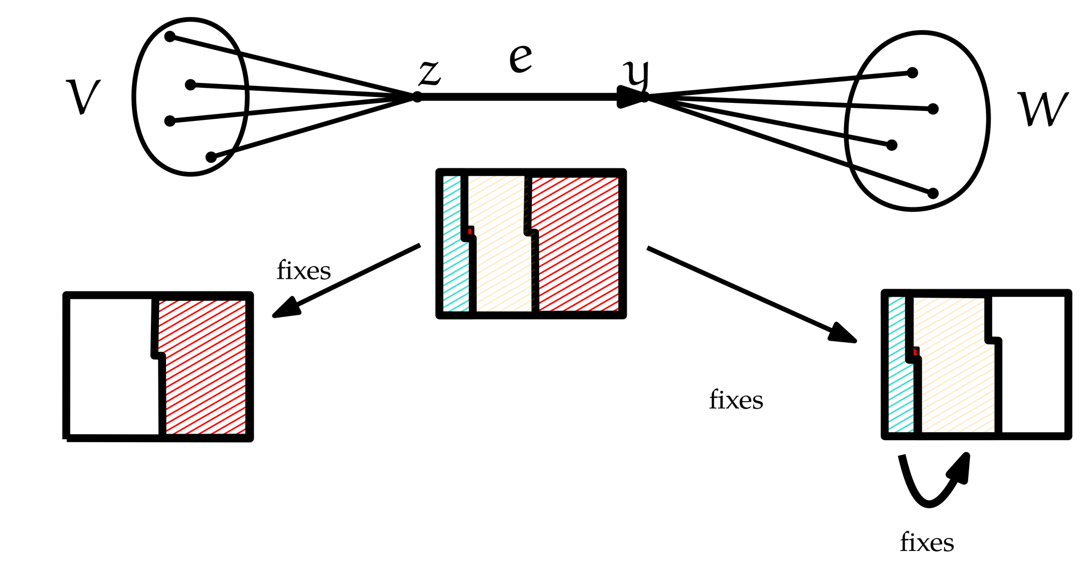


Lemma (Kayibi and Pirzada 2018). *Every edge lies on at most*

$$\#(e) \leqslant N$$

canonical paths where N is the total number of tilings.

Tilings on a canonical path that traverses $e = (z, y)$



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