14. Übungsblatt zur Vorlesung: **Felsner/ Schröder**
Graphentheorie (DS II) 8. Februar 2022
Besprechungsdatum: 14./17. Februar
`http://www.math.tu-berlin.de/~felsner/Lehre/dsII21.html`

This is the last exercise sheet. Thus there are 66 points available in total this semester.

(1) Example graphs:

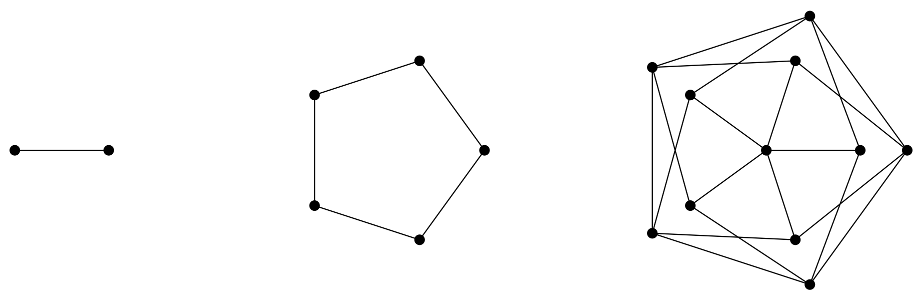
(a) Find a graph G, such that $\chi_f(G) > \frac{n}{\alpha(G)}$.

(b) Find a graph G, such that $\chi_f(G) = \frac{a}{b}$, but $\chi_b(G) > a$.

(2) Bipartite graphs: Let G be a simple graph. Prove:

$$\chi(G) = 2 \Leftrightarrow \chi_f(G) = 2.$$

(3) The Mycielski-graphs obey $M_{k+1} = \mu(M_k)$, where μ is the Mycielski construction:



M_2 M_3 M_4

Figure 1: The Mycielski graphs of chromatic number $2, 3$ and 4

(a) Determine $\chi_f(M_k)$ for $k = 2, 3$ and 4.

(b) Show

$$\chi_f(M_{k+1}) \leq \chi_f(M_k) + \frac{1}{\chi_f(M_k)}.$$

(c) Look for help to prove

$$\chi_f(M_{k+1}) \geq \chi_f(M_k) + \frac{1}{\chi_f(M_k)}.$$

(4) A graph $G = (V, E)$ is *vertex-transitive*, if: For all choices of $u, v \in V$ there is an automorphism $\phi\colon V \to V$ such that $\phi(u) = v$.

(a) Show that Kneser graphs are vertex-transitive.

(b) Let G be a vertex-transitive graph with n vertices. Show:

$$\chi_f(G) = \frac{n}{\alpha(G)}.$$

(c) Find non-vertex-transitive graphs, such that: $\chi_f(G) = \frac{n}{\alpha(G)}$.