13. Übungsblatt zur Vorlesung: **Felsner/ Schröder**
Graphentheorie (DS II) 31. Januar 2021
Besprechungsdatum: 7./10. Februar
`http://www.math.tu-berlin.de/~felsner/Lehre/dsII21.html`

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(1) Planar edge coloring:
A planar edge coloring is a coloring of the edges of a plane graph (planar with a fixed planar drawing) such that any two edges that share a vertex and a face receive different colors. Denote the minimum number of colors in such a coloring by $\chi'_{pl}(G)$.

(a) Prove that if T is a triangulation, then $\chi'_{pl}(T) = 3$.

(b) Find an example of a planar graph G with $\chi'_{pl}(G) > 3$.

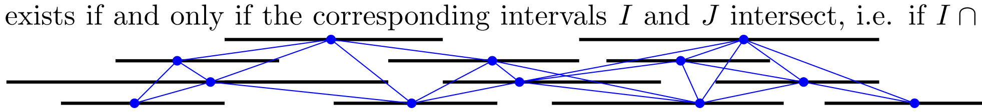
(*) Prove that $\chi'_{pl}(G) \leq 4$ for all 2-connected plane graphs G.
[Remark: You may use that planar graphs are 4-colorable.]

(2) Even more triangle-free graphs with high chromatic number:
Consider the *Double Shift graphs* $S(3, n)$. The vertices of $S(3, n)$ are triples (a, b, c) of integers $1 \leq a < b < c \leq n$ with edges $(a, b, c)(b, c, d)$. Show:

$$\chi(S(3, n)) \geq \log_2 \log_2 n$$

[Hint: Similarly to the shift graphs $S(2, n)$ let $\mathcal{F}_{b,c}$ be the set of colors used for vertices of the kind (a, b, c).]

(3) Let $\mathcal{I}$ be a family of intervals ($\subseteq \mathbb{R}$). We define the intersection graph $G_{\mathcal{I}}$ corresponding to $\mathcal{I}$: For each interval $I \in \mathcal{I}$ there is a vertex v_I and an edge (v_I, v_J) exists if and only if the corresponding intervals I and J intersect, i.e. if $I \cap J \neq \emptyset$.



Show that $G_{\mathcal{I}}$ is perfect.

(4) Complements are perfect graphs / Theorem of Kőnig-Egerváry

(a) Show: G bipartite $\implies \overline{G}$ is perfect.

(b) Show: G bipartite $\implies \overline{\mathcal{L}(G)}$ is perfect.

(5) De Bruijn graphs of Type I/II:
Let $G_n(m)$ be the underlying simple undirected graph of the de Bruijn graph $\mathcal{B}_n(m)$.

(a) Show that for $n > 2$ odd:

$$\chi'(G_n(m)) = \Delta(G_n(m)) \quad (1)$$

(b) Find an m, such that $G_2(m)$ and $G_1(m)$ do not satisfy Equation 1.