12. Übungsblatt zur Vorlesung: Graphentheorie (DS II) — **Felsner/ Schröder**

24. Januar 2022

Besprechungsdatum: 31. Januar/3. Februar

`http://www.math.tu-berlin.de/~felsner/Lehre/dsII21.html`

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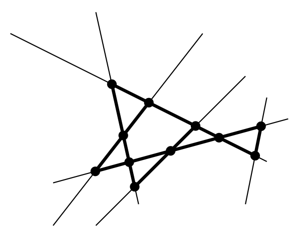
(1) More triangle-free graphs with large chromatic number (Tutte):
Given a triangle-free graph G_k with $\chi(G_k) \geq k$ and n vertices, let G_{k+1} have an independent vertex set X of $k(n-1)+1$ vertices and for all $Y \subset X, |Y| = n$ a copy of G_k, that is connected to Y by a perfect matching. Show that G_{k+1} is triangle-free and that $\chi(G_{k+1}) \geq k+1$.

(2) Greedy coloring worst case:

  (a) Show that there is a bipartite graph on $2n$ vertices and an order of them, such that the Greedy algorithm uses n colors to color it instead of 2.

  (b) Show that there is a planar graph on 2^n vertices and an order of them such that the Greedy algorithm uses more than n colors to color it.

(3) Let there be some lines in the plane, such that no three intersect in a single point. Let the intersection points of these lines be the vertices of a graph G. Two such vertices are adjacent, if they are consecutive on one of the lines. Show that $\chi(G) \leq 3$.



(4) Degeneracy

  (a) Let G be k-degenerate. Prove that

$$\max\{\delta(H) \mid H \text{ subgraph of } G\} \leq k \tag{1}$$

  (b) Now suppose Equation (1) holds. Show that G is k-degenerate.

  (c) What is the largest number of edges a k-degenerate graph can have?

(5) A graph G is *k-chromatic critical* if $\chi(G) = k$, but removing any vertex or edge of the graph leaves the rest $(k-1)$-colorable.

  (a) Find all k-chromatic critical graphs for $k \leq 3$ and an infinite family of such graphs for $k \geq 3$.

  (b) Prove that a triangulation is 3-colorable if and only if it is Eulerian.
  [Hint: Generalize the 3-chromatic critical graphs to 4 in a suitable way.]