11. Übungsblatt zur Vorlesung: **Felsner/ Schröder**
Graphentheorie (DS II) 17. Januar 2022

Besprechungsdatum: 24. /27. Januar
`http://www.math.tu-berlin.de/~felsner/Lehre/dsII21.html`

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(1) A *3-orientation* of a triangulation is an orientation of the inner edges such that any inner vertex has exactly 3 outgoing edges.

(a) Show that there is a 3-orientation for every triangulation.

(b) Show that there are exactly as many 3-orientations of a given triangulation as there are Schnyder Woods.

(c) For every $n \in \mathbb{N}, n \geq 4$, find a triangulation on n vertices that admits exactly 1 Schnyder Wood.

(2) Center Point Theorem:
Show that for any k pointsets $P_1, \ldots, P_k \subset \mathbb{R}^d$, there is a $(k-1)$-dimensional flat F (that is, $F = V + t$, where V is a $(k-1)$-dimensional subspace of $\mathbb{R}^d$ and $t \in \mathbb{R}^d$) such that for every $i \in [k]$, any hyperplane containing F has at most $\frac{d|P_i|}{d+1}$ points of P_i on either side.

(3) The *treedepth* of a graph G is the smallest height of a tree T rooted in r, such that G is a subgraph of

$$(V(T), \{vw | v \text{ is an ancestor of } w \text{ or vice-versa.}\})$$

Prove that a planar graph G on n vertices has treedepth at most $\frac{\sqrt{2n}}{1-\frac{\sqrt{3}}{2}}$.

(4) Prove or disprove:

(a) Let G be a graph with $\chi(G) = k$. Then G has a k-coloring, where one of the color classes has size $\alpha(G)$.

(b) $\chi(G) \leq \bar{d}_G + 1$ for connected G, where $\bar{d}_G = \frac{2|E|}{|V|}$ is the average degree of G.

(c) Every graph G can be colored with $\text{td}(G)$ colors, where td is the treedepth.

(5) A plane simple graph G on $n \geq 4$ vertices is a *quadrangulation*, if every face has degree 4 (even the outer face).

(a) Prove that quadrangulations can be colored with 2 colors.

(b) Prove that any bipartite, planar graph is a subgraph of a quadrangulation.

(c) Prove that any quadrangulation is 2-connected.