7. Übungsblatt zur Vorlesung: Graphentheorie (DS II)

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10. Dezember 2021

Besprechungsdatum: 3. /6. Januar
`http://www.math.tu-berlin.de/~felsner/Lehre/dsII21.html`

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(1) Ramsey Theory
Prove that for any $n \in \mathbb{N}$, there is an $N \in \mathbb{N}$ big enough, such that no matter how you partition $[N]$ into n parts $P_1, \ldots, P_n$, there will be a triple $x, y, z \in P_i$ in one of the parts, such that $z = x + y$.
[Hint: $N = R_2(n; 3, \ldots, 3)$ is big enough.]

(2) Dual graphs of planar graphs

(a) Find a planar simple graph G that is 3-edge-connected, whose dual graph is not unique.

(b) Find two different planar graphs G_1 and G_2 with $G_1^* = G_2^*$.

(c) Let G be a plane graph, that is, a graph drawn without any crossings. Show that if $\kappa(G^*) \geq 2$ then G has at most one component that is not a tree.

(3) A planar simple graph G is a *triangulation*, if a drawing without crossings of G exists, such that every face has degree 3 (even the outer face).

(a) Every simple planar graph G on $n \geq 3$ vertices is a subgraph of a triangulation on n vertices, i.e. G is a *spanning* subgraph of a triangulation.

(b) Every simple planar graph is an induced subgraph of a triangulation.

(c) Triangulations (on more than 3 vertices) are 3-connected.

(4) Euler's formula

(a) Show that for all graphs G with $n \geq 11$ vertices either G or its complement is not planar.

(b) Show, that the property from the lecture, that planar graphs have a vertex of degree ≤ 5, is best possible, by specifying a planar graph without vertices of degree < 5.

(c) Show, that the property from the lecture, that bipartite planar graphs have a vertex of degree ≤ 3, is best possible, by specifying a planar graph without vertices of degree < 3.

(*) Geometric drawings of planar graphs (Christmas bonus exercise)

(a) Show that for all $k \in \{3, 4, 5\}$ every k-gon is star-shaped.
[A set $S \subset \mathbb{R}^n$ is *star-shaped* if a point $p \in S$ exists, such that for all points $q \in S$, the line segment pq is completely contained in S.]

(b) Deduce from this that every triangulation has a planar drawing, such that all edges are line segments.
[Hint: There is a vertex v with $\deg(v) \leq 5$.]

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