**5. Übungsblatt zur Vorlesung:** **Felsner/ Schröder**

**Graphentheorie (DS II)** 22. November 2021

Besprechungsdatum: 29. November/2. Dezember

`http://www.math.tu-berlin.de/~felsner/Lehre/dsII21.html`

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(1) Show the following properties of the de Bruijn graph $\mathcal{B}_n(m)$.

(a) It contains $m$ edge-disjoint arborescences. Are there even more?

(b) Its underlying undirected graph is $m$-edge-connected.

(c) Its underlying undirected graph is $m$-vertex-connected.

(2) A *universal de Bruijn sequence* for $n$ is an infinite sequence of symbols $a_1, a_2, \ldots$ from the infinite alphabet $\mathbb{N}_0$, such that for all $m$ the first $m^n$ symbols form a Memory Wheel, that is a de Bruijn sequence, for words of length $n$ over the alphabet $\{0, \ldots, m-1\}$. Show, that there are universal de Bruijn sequences for all $n \in \mathbb{N}$. *(Hint: Find a way to extend the sequence for the first* $m^n$ *symbols to the first* $(m+1)^n$ *by modeling the problem as a Euler cycle problem on a suitable directed graph.)*

(3) Euler paths and cycles

(a) let $G = (V, E)$ be a connected graph with exactly 2 vertices of odd degree. Show that $G$ admits an Euler path.

(b) Show that in every connected graph, there is a walk that contains every edge exactly twice.

(c) Let $G$ be a Eulerian digraph which is not a directed cycle. Show that $G$ admits an even number of Euler cycles.

(4) How many possibilities are there to draw the "Haus vom Nikolaus" (See Figure 1)?

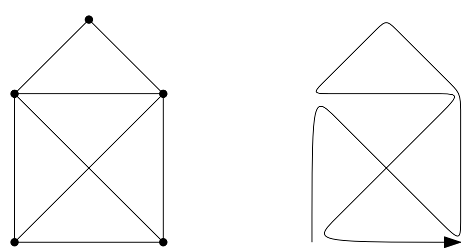

Figure 1: One of the ways of drawing the "Haus vom Nikolaus" without lifting the pen off of the paper while drawing nor drawing any edge twice, a popular German children's game