3. Übungsblatt zur Vorlesung: **Felsner/ Schröder**
Graphentheorie (DS II) 8. November 2021
Besprechungsdatum: 15. /18. November
`http://www.math.tu-berlin.de/~felsner/Lehre/dsII21.html`

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(1) Let $K_{m,n}$ be the complete bipartite graph with parts $\{1', \dots, m'\}$ as well as $\{1, \dots, n\}$.
[Remark: This exercise gives 2 points.]

(a) How many spanning trees does $K_{2,n}$ have? How many non-isomorphic ones?

(b) How many spanning trees does $K_{3,n}$ have? How many non-isomorphic ones?
[This is a rounded polynomial but a not fully simplified sum is acceptable.]

(c) Let $m \leq n$. How many spanning trees does $K_{m,n}$ have?
[Hint: Clarke's proof of the Cayley formula can be adapted to $K_{m,n}$.]

(2) G is a *Laman graph*, if it has $2|V(G)| - 3$ vertices and all of its subgraphs H with at least 2 vertices have at most $2|V(H)| - 3$ edges.

(a) Show that every Laman graph can be obtained from K_2 by a sequence of so-called Henneberg steps: Either add a vertex of degree 2 adjacent to any two vertices of the graph (H_1) or replace an edge connecting two vertices u and v by a vertex x adjacent to u, v and any third vertex (H_2).

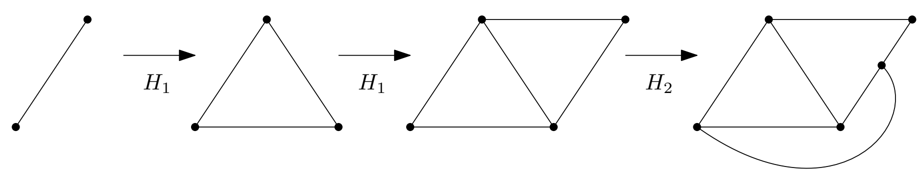

Figure 1: A graph constructed by Henneberg steps H_1 and H_2

(b) Prove that in every Laman graph, there are two spanning trees that share exactly one edge.

(3) Let $d_1, ..., d_n \in \mathbb{N}$ such that their sum is $2n - 2$. From exercise (2a) on sheet 2 we know that $(d_1, ..., d_n)$ is the degree sequence of a tree, if and only if $\sum d_i = 2n - 2$. Show that there are

$$\frac{(n-2)!}{\prod_i (d_i - 1)!}$$

trees with vertex set $[n]$, such that vertex i has degree d_i.
[We clearly do not ask for isomorphism classes here.]

(4) A connected graph with at most one simple cycle is called a *pseudotree*. Prove that any two of these properties are equivalent:

- G is a pseudotree.
- G has n edges.
- There is an edge $e \in E(G)$ such that $G - e$ is connected.