**1. Übungsblatt zur Vorlesung: Graphentheorie (DS II)**

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`http://www.math.tu-berlin.de/~felsner/Lehre/dsII21.html`

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(1) Degrees

(a) Does a simple graph with at least 2 vertices exist, such that each vertex has a different degree?

(b) A *k–regular* graph is a graph, whose vertices all have degree $k$. How many 3–regular graphs with 1,2,. . . ,8 vertices are there?

(2) Circles: Let $G = (V, E)$ be such that $V = [n] := \{1, ..., n\}$. Prove the following:

(a) If $G$ contains a **circle**, then $G$ contains a **simple cycle**.
[A circle is a closed walk (it ends up in the same vertex as it started, but can use edges and vertices multiple times), a simple cycle is a circle, that doesn't use any vertex or edge twice.]

(b) If $G$ contains a circle of odd length, then $G$ contains a simple cycle of odd length.

(c) If $G$ contains no simple cycles of even length, then $|E| \leq \frac{3}{2}(n-1)$.

(3) Basic facts of graph theory

(a) Prove or disprove: If $u \in V$ and $v \in V$ are the only vertices of odd degree in $G$, then you can walk from $u$ to $v$ in $G$.

(b) Prove that $H_d$, the $d$–dimensional hypercube, is bipartite by presenting a suitable homomorphism $H : H_d \to K_2$, where $K_t$ is the complete graph with $t$ vertices.

(c) Prove that the following is equivalent for a graph $G$:

- $G$ is bipartite
- The vertex set of $G$ can be partitioned into sets $X$ and $Y$, that both induce an *empty* graph (a graph without edges).
- $G$ contains no odd circles.

(4) Trigger problems

(a) Given a point set $P$ in the plane, if $d$ is the maximal distance between two points in $P$, up to how many pairs of points can have distance $d$?

(b) Present a collection $A_1, ..., A_{n+1} \subset [n]$ of distinct sets, such that no $x \in [n]$ exists, such that $A_1 + x, ..., A_{n+1} + x \subset [n]$ are still distinct.

(c) Prove that in any collection $A_1, ..., A_{n-1} \subset [n]$ of distinct sets, some $x \neq y \in [n]$ exist, such that $A_1 + x + y, ..., A_{n-1} + x + y \subset [n]$ are still distinct.