

1. Practice sheet for the lecture:  
**Graph Theory (DS II)**

Felsner/ Schröder  
 30. September 2019

Due dates: 24./25. October

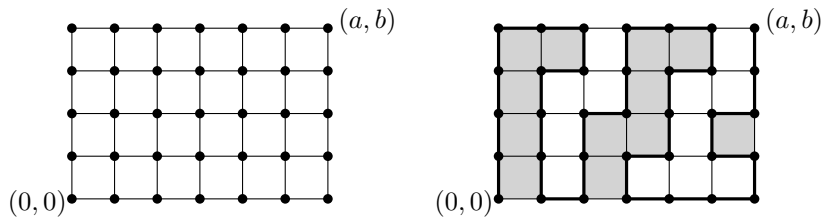
<http://www.math.tu-berlin.de/~felsner/Lehre/dsII19.html>

- (1) Two thieves have stolen a necklace with  $2m$  golden and  $2n$  silver beads. Show that the necklace can be cut into two pieces such that each piece consists of  $m$  golden and  $n$  silver beads. (Independent of the ordering of the beads!)

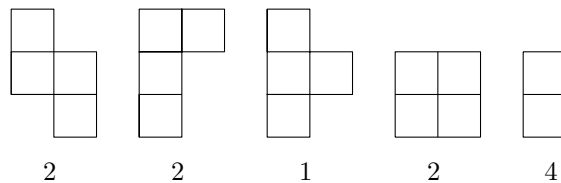
Why are there two points on the equator with the same temperature? What is the connection between these two questions?

- (2) Consider a planar graph with vertices on integer coordinates  $0 \leq x \leq a$ ,  $0 \leq y \leq b$  and edges between pairs of vertices with distance 1, see the figure below. Let  $P$  be a path from  $(0, 0)$  to  $(a, b)$  visiting all vertices exactly once.  $P$  partitions the set of boxes into two sets: the set reachable from top or left and the set reachable from bottom or right. Show that the sets have the same cardinality.

In the right figure the set of boxes reachable from bottom and right is shaded.



- (3) Ralf and Anna go to a dinner party with  $n - 1$  other couples. Each participant shakes hands with every unknown person. Afterwards Ralf does a survey and discovers that everyone of the  $2n - 1$  other attendees shook hands with a different number of people. How many people did Anna shake hands with?
- (4) Show that there is no covering of the  $6 \times 6$  chess board with the following 11 pieces. The multiplicities of the pieces are given by the numbers in the picture.



- (5) Given are  $n$  points on a circle. Draw a segment between each pair of points. In how many regions is the disk cut? (Assume that the  $n$  points are positioned such that no three segments intersect in one point.)

