Übungsblatt "Graphs, Order, and Geometry" **SoSe 2024**

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(1) The Erdős–Szekeres lemma can be stated as follows: If $n = ab + 1$ and $x_1, \ldots, x_n$ is a sequence of n numbers, then the sequence contains a strictly increasing subsequence of $a + 1$ terms or a weakly decreasing subsequence of $b + 1$ terms.
Prove the lemma and think about generalizations.

(2) Describe an algorithm to find a maximum weight free set in a weighted binary tree, here a set of nodes is called free if no two of them are on a joint path to the root.

(3) A geometric graph is a graph given with a straight line drawing. A convex geometric graph has its vertices in convex position, e.g. as points on a circle. Show hat a convex geometric graph with n vertices and no three pairwise crossing edges has at most $4n - 10$ edges.

(4) Let G be a geometric graph with n vertices such that each edge is crossed by at most one other edge. How many edges can G possess?

(5) Consider intersection graphs of discs and the problems Chromatic Number and Maximum Independent Set. Develop approximation algorithms.

(6) As before but this time for intersection graphs of translates of a fixed convex shape.