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**Topology**

Winter term 2021/2022

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**Exercise Session Sheet 11**

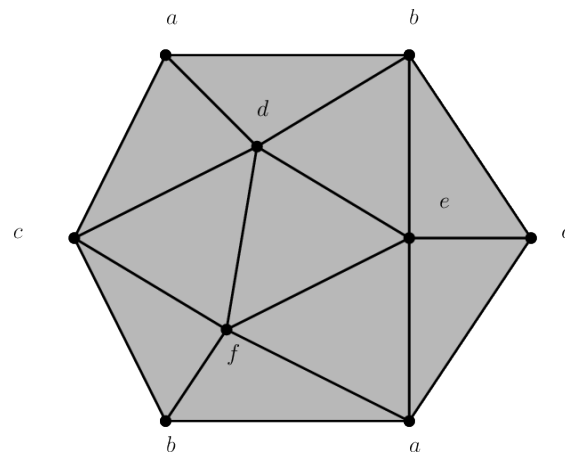
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**Exercise 1**

Let  $P \subset \mathbb{R}^3$  be a 3-dimensional polytope. Show that the fundamental group  $\pi_1(\partial P)$  of the boundary of  $P$  is trivial.

**Exercise 2**

Compute the fundamental group  $\pi_1(K)$  and the Euler characteristic  $\chi(K)$  of the following space  $K$ :

**Exercise 3**

Compute the fundamental group of the figure eight topological space using a triangulation.

**Exercise 4**

Compute the fundamental group of the figure eight topological space using van Kampen's theorem.

**Exercise 5**

Compute the fundamental group of the projective plane using van Kampen's theorem on the *polygon model* of the projective plane.

**Exercise 6**

Showcase van Kampen's theorem on the polygon model for any closed surface. Show that the polygon model of the Klein bottle and the *usual* model of the Klein bottle describe the same space.

**Exercise 7**

Give the emergency triangulation of the projective plane, compute its Euler characteristic and show how surgery works.

**Exercise 8**

Let  $X$  be a topological space and  $f: \mathbb{S}^{n-1} \rightarrow X$  be a map. Compute  $\pi_1(\mathbb{D}^n \cup_f X)$ .