Topology Monday, 17/1/2022

Winter term 2021/2022

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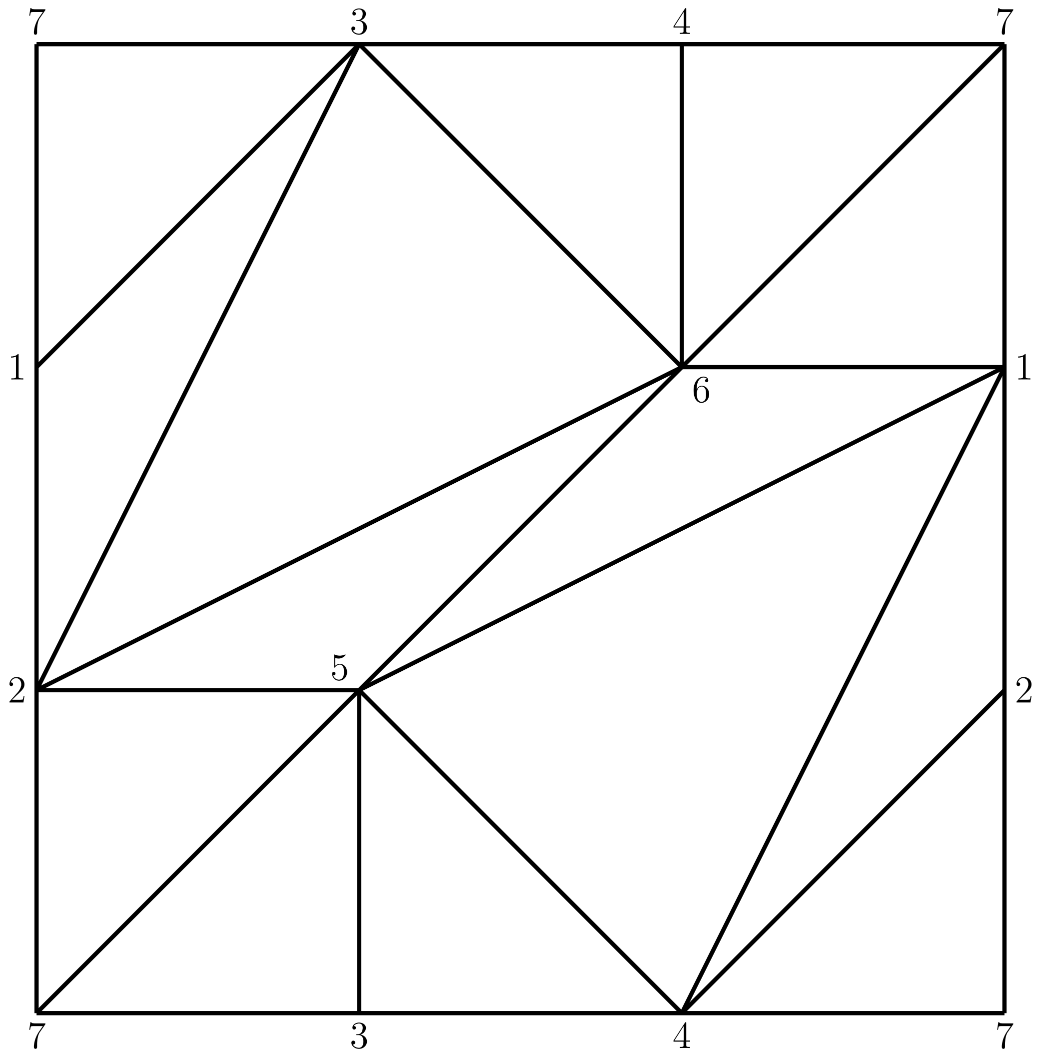
Exercise Sheet 10

Due-date: Wednesday, 26/1/2022, *before* the lecture starts.

Exercise 1 **10 Points**

Consider the following simplicial complex K where all triangles are filled.



a) Compute the Euler characteristic $\chi(|K|) := \chi(K) := \#V(K) - \#E(K) + \#F(K)$ of $|K|$, where $V(K)$ is the set of vertices, $E(K)$ is the set of edges and $F(K)$ is the set of 2 - dimensional faces of K.

b) Compute the fundamental group $\pi_1(|K|, v_0)$ of $|K|$ for some vertex $v_0 \in |K|$.

c) If possible, give an orientation of K.

d) Let L be a triangulation of a polyhedral surface S. Show that following holds:

$$\#V(L) \geq \frac{7 + \sqrt{49 - 24\chi(S)}}{2} \quad .$$

What does this mean for the complex K?

e) Choose and draw a simple closed polygonal curve L in $|K|$, i.e. a curve L consisting of edges of $|K|$, that **doesn't** separate $|K|$. Pass to the second barycentric subdivision K^2 of K and take all simplices of K^2 meeting L in order to obtain a *thickenning* N of L. Draw N and show that it is homeomorphic to a cylinder. Compute $\chi(N)$.

f) Let M be the subcomplex of K^2 consisting of exactly those simplices of K^2, which do not meet L. According to e), the boundary complex of M consists of two components L_1 and L_2, each triangulating a circle. Recall the definition of the cone complex CL_i and set

$$K_* := M \cup CL_1 \cup CL_2 \quad .$$

Draw as much as you can and compute $\chi(K_*)$.

Exercise 2 **4 Points**

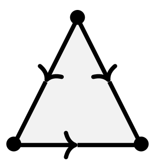
Give two triangulations K_1 and K_2 of the Klein bottle and simple closed polygonal curves L_i in $|K_i|$, such that

a) the thickenning N_1 of L_1 is homeomorphic to a cylinder.

b) the thickenning N_2 of L_2 is homeomorphic to a Möbius strip.

Exercise 3 **4 Points**

Use van Kampen's Theorem (cf. Exercise Session) in order to compute the fundamental group of the dunce hat. Is it orientable?



Exercise 4 **2 Points**

Let K be an *abstract* simplicial complex. Show that changing the orientation of a simplex $\sigma \in K$ changes the orientation of every face of σ.