Discrete Geometry II Thursday, 26/11/2020
Winter term 2020/21
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Exercise sheet 4

Due-date: Friday, 04/12/2020, via e-mail to eble@math.tu-berlin.de

Exercise 1 **4 Points**

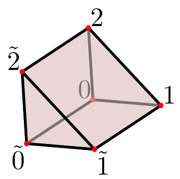
Let $l, i > 0$ be two fixed integers. Show that there is a unique $j \in \mathbb{N}$ and a unique sequence of natural numbers $n_i > n_{i-1} > \cdots > n_j \geq j \geq 1$ such that

$$l = \binom{n_i}{i} + \binom{n_{i-1}}{i-1} + \cdots + \binom{n_j}{j} \ .$$

Exercise 2 **4 Points**

a) Consider all subsets of $\mathbb{N}$ with cardinality k. Show that the *reverse lexicographic order* on those sets (i.e. $G < H \iff \max(G \setminus H) < \max(H \setminus G)$) is a linear order.

b) Let $P \subset \mathbb{R}^3$ be the following triangular prism:



Consider the triangulation $\mathcal{T} := \{\{0, 1, 2, \tilde{0}\}, \{1, 2, \tilde{0}, \tilde{1}\}, \{2, \tilde{0}, \tilde{1}, \tilde{2}\}\}$ of P. Compute the f-vector of $\mathcal{T}$ and construct the compressed simplicial complex Δ_f.

Exercise 3 **5 Points**

a) Determine the h-vector of the cubes in all dimensions.

b) Determine the h-vector of the cross polytopes in all dimensions.

c) Determine the h-vector of the permutahedra in all dimensions.

Exercise 4 **3 Points**

Let P be a simplicial d-polytope and ∂P its boundary complex. In general, ∂P is always (homeomorphic to) some sphere. In this special case, ∂P is a simplicial $(d-1)$-sphere. Show that $h_d(\partial P) = 1$.