Discrete Geometry II
Winter term 2020/21
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Session 3

Very basics of topology

Exercise 1
Show that the projection $\pi\colon \mathbb{S}^n \to \mathbb{RP}^n$ is an open map.

Exercise 2
Show that a triangulable space is a compact metric space.

Exercise 3
Give the f-vector for the triangulation of space A.

Exercise 4
Triangulate space B.

Very basics of commutative algebra

Exercise 5
Give an example of a finitely generated R-module which is not free. Is $\mathbb{Z}$ a $\mathbb{Z}/m\mathbb{Z}$-module for some $m \in \mathbb{Z}$?

Exercise 6
Show that if an ideal I contains a unit, then I is trivial.

Exercise 7
Do you know how ideals of $\mathbb{C}[X]$, $\mathbb{R}[X]$ and $\mathbb{Z}[X]$ look like? What do you know about the ring $K[X_1, \ldots, X_n]$ for K an algebraically closed field?

Exercise 8
Show that a finitely generated K-algebra is the quotient of a polynomial ring over K.

Very very basics of algebraic geometry

Exercise 9
Let L be a linear space. Write L as $V(f_1, \ldots, f_k)$.

Exercise 10
Let K be a field. Show that $f \in K[X]$ has only finitely many zeros.

Exercise 11
Draw $V(x^3 - x - y^2) \subset \mathbb{R}^2$.