Discrete Geometry II
Winter term 2020/21
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Session 2

Let P be a polytope throughout.

Exercise 1
Let Q_1 be a bipyramid over the 3-simplex. Let Q_2 be a pyramid over the bipyramid over the 2-simplex. Determine $\dim(Q_i)$. Draw $G(Q_i)$. You may even be able to draw Q_i. Determine the f-vectors of Q_i. It is even possible to read off the face lattices $\mathcal{F}(Q_i)$. Show that Q_1 and Q_2 have isomorphic graphs but are not combinatorially isomorphic.

Exercise 2
Let v be a vertex of P and let H^* be a closed halfspace with bounding hyperplane H such that $v \in H$. Assume that all edges of P which contain v are contained in H^*. Prove that $P \subset H^*$. Thus, H is a supporting hyperplane for P.

Exercise 3
Let H be any hyperplane and $W \subset H \cap \mathrm{vert}(P)$ be a *proper* subset. Show that removing W from $G(P)$ leaves a connected subgraph.
Hint: Balinski flavoured proof...

Exercise 4
Make the 'beyond-$F^{'}$' statement of the total separability degree theorem precise: Why is $(G(P) \setminus V^{'})|_{X^{'}}$ connected? Here $X^{'}$ denotes the vertices of P lying beyond the facet $F^{'}$ of $P^{'}$.

Exercise 5 (guess work)
The g-Theorem is a deep statement about simplicial polytopes and their f-vectors. What do you think is the situation for arbitrary polytopes? Is it a tiny/small/big/huge step to go there?