trivial I-cocycles β_n with values in the irreducible representations π_n, acting on the spaces H_n, such that $\|\beta(g)\| = \lim_{n\to\infty} \|\pi_n(g)h_n - h_n\| = \lim_{n\to\infty} \|\beta_n(g)\| \quad (h_n \in H_n)$.

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LANDAU–LIFSHITS EQUATION. "DRESSING UP" PROCEDURE. ELEMENTARY EXCITATIONS

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The "dressing up" procedure (in the sense of V. E. Zakharov–A. B. Shabat) is applied to construct solitons and breathers for the Landau–Lifshits equation.

1. The Landau–Lifshits equation

$$\vec{S}_t = [\vec{S}\times\vec{S}_{xx}] + [\vec{S}\times J\vec{S}], \quad S_1^2+S_2^2+S_3^2=1,$$
$$J = \mathrm{diag}(J_1, J_2, J_3), \quad J_1 < J_2 < J_3 \tag{1}$$

describes the nonlinear dynamics of anisotropic ferromagnetics in the absence of an external field.

Sklyanin [1] and Borovik [2] established independently that (1) is the compatibility condition for the system of linear equations:

$$\frac{\partial\Psi}{\partial x} = U\Psi, \quad \frac{\partial\Psi}{\partial t} = V\Psi, \quad U = -i\sum_{\alpha=1}^{3} S_\alpha W_\alpha(u)\sigma_\alpha,$$
$$V = 2iW_1(u)W_2(u)W_3(u)\sum_{\alpha=1}^{3} S_\alpha\sigma_\alpha W_\alpha^{-1}(u) - i\sum_{\alpha=1}^{3} W_\alpha(u)\sigma_\alpha[\vec{S}\times\vec{S}_x]_\alpha,$$
$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \tag{2}$$
$$W_1(u) = \rho\frac{1}{\mathrm{sn}(u,k)}, \quad W_2(u) = \rho\frac{\mathrm{dn}(u,k)}{\mathrm{sn}(u,k)}, \quad W_3(u) = \rho\frac{\mathrm{cn}(u,k)}{\mathrm{sn}(u,k)},$$
$$k = \sqrt{\frac{J_2-J_1}{J_3-J_1}}, \quad \rho = \frac{1}{2}\sqrt{J_3-J_1}, \quad W_\alpha^2 - W_\beta^2 = -\frac{1}{4}(J_\alpha - J_\beta).$$

Sklyanin also showed that (1) describes a completely integrable Hamiltonian system, and calculated an infinite series of commuting integrals of motion. However, until recently one was able to get exact solutions of the Landau–Lifshits equation either by direct methods, not using the algorithm of the method of the inverse problem at all [3, 4], or by methods which used this algorithm only partially. As to the latter, we have in mind [5], where multisoliton solutions of (1) are constructed by the method of Hirota, and [6], where the same solutions are found with the help of the theory of free fermion fields.

The difficulties in applying the method of the inverse problem to (1) were connected with the absence of an exact formulation of the matrix Riemann problem which should correspond to the elliptic bundle (2). The formulation of the Riemann problem needed was found quite recently by A. V. Mikhailov and was investigated by him and also Rodin in [7, 8]. In

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particular, A. V. Mikhailov proposed an algorithm for getting soliton solutions of the Landau–Lifshits equation.

In the present paper, repelled by the Riemann problem as formulated by A. V. Mikhailov, an alternative procedure for "dressing up" to that proposed in [7] is developed. The method which we propose is the transfer to the case of a torus of the method originally developed for rational bundles in [9, 10], and differs from it, in our view, in its great simplicity and clarity. We note in addition that our approach allows us to easily distinguish and classify all elementary excitations corresponding to the completely anisotropic model of ferromagnetics (1).

As in the rational case, the dressing up procedure for the Riemann problem we consider is closely connected with the Darboux transformations for the Landau–Lifshits equation. The recent paper of Cherednik [11] is devoted to the general theory of these transformations for elliptic bundles. The essential difference of the methods of [11] from the elementary approach we propose is the nontrivial use of ideas of algebraic geometry. The latter circumstance, being an absolute advantage from a general-theoretical point of view, strongly complicates the concrete calculations of solutions of (1) by the method of I. V. Cherednik.

2. We begin with the formulation of the regular Riemann problem for (1).

Mikhailov's THEOREM. Let $\Psi(u)$ be a doubly periodic invertible matrix function ($4K$, $4iK'$ being the periods of the Jacobi elliptic functions of modulus k), which is holomorphic outside the point $\{0\}$ ($\{u\}=\{u, u+2K, u+2iK', u+2K+2iK'\}$) and which has the following properties:

$$\Psi(u) \underset{k\to\infty}{=} (\Phi+\Phi_1 k^{-1}+\dots)\exp(-ikx\sigma_3+2ik^2t\sigma_3); \tag{3}$$

the asymptotic expansion for $u\sim 0$ / k^{-1} is the local parameter;

$$\sigma_3\Psi(u+2K)\sigma_3=\Psi(u) \tag{4}$$

$$\sigma_1\Psi(u+2iK)\sigma_1=\Psi(u) \tag{5}$$

$$\det\Psi(u) \text{ is independent of } x \text{ and } t. \tag{6}$$

(We note that it follows from the reductions (4-5) that the function $\Psi(u)$ has essential singularities at all points of $\{0\}$.) On the torus there is given a system of contours Γ_i and conjugation matrices $G_i(u)$;

$$\Psi_-(u)=\Psi_+(u)G_i(u)\big|_{u\in\Gamma_i} \text{ is the conjugation problem on } \Gamma_i \tag{7}$$

The contours Γ_i and matrices $G_i(u)$ must be chosen so that (7) does not contradict the reductions (4-5).

Then by (3-7) the function Ψ can be uniquely determined, and $\vec{S}=(S_1,S_2,S_3)$ such that

$$\sum_\alpha S_\alpha\sigma_\alpha=\Phi\sigma_3\Phi^{-1} \tag{8}$$

is a solution of the Landau–Lifshits equation (1). In case the reduction

$$\sigma_2\overline{\Psi(\bar u)}\sigma_2=\Psi(u) \tag{9}$$

also holds, the vector $\vec{S}$ has real coordinates.

Now we formulate the Riemann problem with zeros.* In this case zeros of $\det\Psi(u)$ at fixed points $u_1,\dots,u_{8N}$ (independent of x and t) are admitted, while the function $\Psi(u)$ must, in a neighborhood of u_i, have the form:

$$\Psi(u)\underset{u\sim u_i}{=}\hat\Psi(u)\begin{pmatrix}u-u_i & 0\\ 0 & 1\end{pmatrix}C_i, \tag{10}$$

where C_i is an invertible matrix, independent of x,t,u, $\hat\Psi(u)$ is holomorphic and invertible in a neighborhood of u_i. Condition (3) is now replaced by the requirement

$$\Psi(u)\underset{k\to\infty}{=}(\Phi+\Phi_1k^{-1}+\dots)\exp(-ikx\sigma_3+2itk^2\sigma_3)k^N, \tag{11}$$

which guarantees the equality of the number of zeros and the number of poles of $\det\Psi$.

3. We describe the method of "dressing up." Suppose we have a function $\Psi_0(u)$ satisfying (3-9) or (4-11) (in what follows, this will be the function

$$\Psi_0(u)=\exp(-ix\sigma_3W_3(u)+2itW_1(u)W_2(u)\sigma_3),$$

corresponding to $\vec{S}=(0,0,1)$, although the method given can "dress up" any solution discovered). We shall seek a new solution of the problem (4-11) in the form $\Psi=f(u,x,t)\Psi_0$, where f is a matrix-valued function satisfying two conditions:

a) it satisfies the reductions (4-5, 9),

b) its matrix elements have poles only at points which are essential singularities of the function Ψ_0, where the pole is of the M-th order.

These conditions give the following form for the function f / $M=1$ /:

$$f=(iaW_1(u)\sigma_1+ibW_2(u)\sigma_2+icW_3(u)\sigma_3+d), \tag{12}$$

where a,b,c and d are real-valued functions of x and t, which we find from the condition that (10) holds.

(10) can be rewritten (cf. [10]) in the more convenient form

$$\Psi(u_i)\begin{pmatrix}1\\ c_i\end{pmatrix}=0, \tag{13}$$

where c_i is independent of x,t,u. Now we shall see where the zeros of $\det\Psi=\det f$ are. Let $\det\Psi(u_0)=0$; then it follows from (4-5) that $\{u_0\}$ are zeros of $\det\Psi$. Moreover, it follows directly from the form of (12) that $\det f$ is an even function in u so $\{-u_0\}$ are also zeros of $\det\Psi$. Thus, we know all 8 zeros of the function $\det\Psi$. In order that (10) be valid at the points of $\{u_0\}$, $\{-u_0\}$, it is necessary to require that

$$\Psi(u_0)\begin{pmatrix}1\\ A\end{pmatrix}=0,\ \Psi(-u_0)\begin{pmatrix}1\\ B\end{pmatrix}=0. \tag{14}$$

(14) is a linear homogeneous system of four equations in the four unknowns a,b,c,d. Since $\det f$ is even, the rank of this system is equal to 3. Thus, (14) completely deter-

*Starting here we stop following A. V. Mikhailov and start treating the zeros of $\det\Psi$ as "regular singularities of the Ψ-function" in the spirit of [10].

mines the quantities of interest to us $a/d, b/d$, and c/d. It is clear that there are no problems connected with the concrete calculations involving (14).

The expressions for the coordinates of vector $\vec{S}$ follow from (8) and (12):

$$S_1 = \frac{2ca}{c^2+a^2+b^2}, \quad S_2 = \frac{2bc}{c^2+a^2+b^2}, \quad S_3 = \frac{c^2-a^2-b^2}{c^2+a^2+b^2}. \tag{15}$$

We note that now we do not have to be concerned about satisfying (6) in getting the expressions for $\vec{S}(x,t)$, since it can always be made to hold by a simple renormalization of the Ψ-function with respect to which the vector $\vec{S}$ is invariant.

In order that (9), which guarantees that $\vec{S}$ is real, hold, we must require that the set of zeros of $\det\Psi$ be invariant with respect to complex conjugation. This condition permits us to find all possible positions of zeros (we note right away that if $\operatorname{Im} u_0 = 0$ it is impossible to get real a, b, c, d).

Case I. $\operatorname{Im} u_0 = K', -K'$; $W_1(u_0) \in \mathbb{R}$, $\operatorname{Re} W_2(u_0) = \operatorname{Re} W_3(u_0) = 0$.

1) We set $B = 0, A = \pm i e^{\rho}$, $\rho \in \mathbb{R}$. From (14, 15) we get the well-known (cf. [1]) expressions for solitons and antisolitons

$$S_1 = \pm \frac{2iW_2(u_0)}{\sqrt{J_2-J_1}\, ch(\xi x - \tau t - \Delta)}, \quad S_2 = \mp \frac{2W_1(u_0)}{\sqrt{J_2-J_1}\, ch(\xi x - \tau t - \Delta)},$$

$$S_3 = th(\xi x - \tau t - \Delta), \quad \xi = 2iW_3(u_3), \quad \tau = 4iW_1(u_0)W_2(u_0). \tag{16}$$

2) $A = iA_0, B = iB_0$; $A_0, B_0 \in \mathbb{R}$, $A_0 B_0 < 0$.

In this case we get a solution with momentum zero describing the process of collision of two solitons

$$S_1 = \pm \frac{2\mathcal{D}\, sh\,\tau(t-t_0)\, sh\,\xi(x-x_0)}{sh^2\xi(x-x_0) + E^2 + F^2 sh^2\tau(t-t_0)}$$

$$S_2 = \mp \frac{2E\, ch\,\tau(t-t_0)\, sh\,\xi(x-x_0)}{sh^2\xi(x-x_0) + E^2 + F^2 sh^2\tau(t-t_0)} \tag{17}$$

$$S_3 = \frac{sh^2\xi(x-x_0) - E^2 - F^2 sh^2\tau(t-t_0)}{sh^2\xi(x-x_0) + E^2 + F^2 sh^2\tau(t-t_0)},$$

where $E = \frac{W_3}{W_2}(u_0), \mathcal{D} = \frac{iW_3}{W_1}(u_0), F = \frac{1}{2}\sqrt{J_2-J_1}\, \frac{W_3}{W_1 W_2}(u_0)$.

3) $A = iA_0, B = iB_0$; A_0; $B_0 \in \mathbb{R}$, $A_0 B_0 > 0$

This is a collision of a soliton with an antisoliton

$$S_1 = \pm \frac{2\mathcal{D}\, ch\,\tau(t-t_0)\, ch\,\xi(x-x_0)}{ch^2\xi(x-x_0) + \mathcal{D}^2 + F^2 sh^2\tau(t-t_0)}$$

$$S_2 = \mp \frac{2E\, sh\,\tau(t-t_0)\, ch\,\xi(x-x_0)}{ch^2\xi(x-x_0) + \mathcal{D}^2 + F^2 sh^2\tau(t-t_0)} \tag{18}$$

$$S_3 = \frac{ch^2\xi(x-x_0) - \mathcal{D}^2 - F^2 sh^2\tau(t-t_0)}{ch^2\xi(x-x_0) + \mathcal{D}^2 + F^2 sh^2\tau(t-t_0)}.$$

The solutions corresponding to the two other possible positions of the Ψ-function will be breathers. For them we give the final expressions.

Case II. $\operatorname{Re} u_0 = K, -K$; $W_1(u_0), W_2(u_0) \in \mathbb{R}$, $\operatorname{Re} W_3(u_0) = 0$, $\overline{A}B = 1$

$$S_1 = -\frac{2H \sin\omega(t-t_0)\, ch\,\xi(x-x_0)}{ch^2\xi(x-x_0) + H^2 + K^2\cos^2(t-t_0)},$$

$$S_2 = -\frac{2G \cos\omega(t-t_0)\, ch\,\xi(x-x_0)}{ch^2\xi(x-x_0) + H^2 + K^2\cos^2\omega(t-t_0)}, \tag{19}$$

$$S_3 = \frac{ch^2\xi(x-x_0) - H^2 - K^2\cos^2\omega(t-t_0)}{ch^2\xi(x-x_0) + H^2 + K^2\cos^2\omega(t-t_0)},$$

where $H = \frac{iW_3}{W_1}(u_0), G = \frac{iW_3}{W_2}(u_0), K = \frac{\sqrt{J_2-J_1}}{2}\, \frac{iW_3}{W_1 W_2}(u_0)$, $\quad \xi = 2iW_3(u_0), \omega = 4W_1 W_2(u_0)$.

Case III. $\operatorname{Re} u_0 = 0, 2K$; $\operatorname{Re} W_1(u_0) = \operatorname{Re} W_2(u_0) = \operatorname{Re} W_3(u_0) = 0$, $\overline{A}B = -1$.

$$S_1 = -\frac{2L \cos\omega(t-t_0)\, sh\,\xi(x-x_0)}{sh^2\xi(x-x_0) + L^2 + N^2\sin^2\omega(t-t_0)},$$

$$S_2 = \frac{2M \sin\omega(t-t_0)\, sh\,\xi(x-x_0)}{sh^2\xi(x-x_0) + L^2 + N^2\sin^2\omega(t-t_0)}, \tag{20}$$

$$S_3 = \frac{sh^2\xi(x-x_0) - L^2 - N^2\sin^2\omega(t-t_0)}{sh^2\xi(x-x_0) + L^2 + N^2\sin^2\omega(t-t_0)},$$

where $L = \frac{W_3}{W_1}(u_0), M = \frac{W_3}{W_2}(u_0), N = \frac{i\sqrt{J_2-J_1}}{2}\, \frac{W_3}{W_2 W_1}(u_0)$, $\quad \xi = 2iW_3(u_0), \omega = 4W_1W_2(u_0)$.

4. Up to now we have considered the dressing up connected with the addition of 8 zeros of the Riemann problem. Obviously if we want to add $8n$ zeros, then we should apply the procedure described above n times with different $A_1, \ldots, A_n, B_1, \ldots, B_n$. Thus, we can get the n-soliton solution ($\operatorname{Re} A_k = B_k = 0, k = 1, \ldots, n$) and also solutions describing the interaction of the different elementary excitations considered in paragraph 3. We note that it is impossible to choose the zeros of the corresponding Riemann problem arbitrarily, but all of them have to lie on the collection of lines described.

The construction of a set of zeros which does not split up into the subsets considered in paragraph 3 is possible for "two-soliton dressing up" (the case of a pole of f at $\{0\}$ of order $M = 2$). We shall dwell on this result in more detail. Here

$$f(u,x,t) = i\sigma_1(aW_1(u) + bW_2(u)W_3(u)) + i\sigma_2(cW_2(u) + dW_1(u)W_3(u)) + i\sigma_3(eW_3(u) + fW_1(u)W_2(u)) + (g + hW_3^2(u)), \tag{21}$$

and the requirement that all the functions be real is preserved. The condition that $\det f$ be even in u is also preserved, from which it follows that

$$ab + cd + ef = 0. \tag{22}$$

Then if u_1 and u_2 are two zeros of the Riemann problem, one has that $\{u_1\}, \{u_2\}, \{-u_1\}, \{-u_2\}$ are also zeros, so in all there are 16 zeros. The conditions

$$\Psi(u_1)\begin{pmatrix}1\\A_1\end{pmatrix}=0,\ \Psi(u_1)\begin{pmatrix}1\\A_2\end{pmatrix}=0,\ \Psi(-u_1)\begin{pmatrix}1\\B_1\end{pmatrix}=0,\ \Psi(-u_2)\begin{pmatrix}1\\B_2\end{pmatrix}=0 \tag{23}$$

guarantee us form (10) for the function $\Psi(u)$ at the zeros of $\det\Psi(u)$. Solving (22-23) we get expressions for the functions b, d, f, h and hence also for

$$S_1=\frac{2(dh+bf)}{h^2+f^2+d^2+b^2},\quad S_2=\frac{2(bh-df)}{h^2+f^2+d^2+b^2},\quad S_3=\frac{h^2+f^2-d^2-b^2}{h^2+f^2+d^2+b^2} \tag{24}$$

In order that (9) hold it is necessary that the set of zeros $\{u_1\},\{u_2\},\{-u_1\},\{-u_2\}$ be invariant with respect to complex conjugation. This reduces either to the conditions on u_1 and u_2, which were described in paragraph 3, or to the equation

$$\{\bar{u}_1\}\cup\{-\bar{u}_1\}=\{u_2\}\cup\{-u_2\}.$$

In the latter case it is also easy to get the restrictions on A_1, A_2, B_1, B_2; for example, if $u_2=\bar{u}_1$, then $A_1\bar{A}_2=-1$, $B_1\bar{B}=-1$. Thus, the solution obtained will be characterized by three arbitrary complex parameters u_1, A_1, B_1. It is obvious that in view of the arbitrariness of u_1, it is impossible to get the double application of "dressing up" described in paragraph 3.

Now we shall show how to get a solution which describes a breather with nonzero momentum. For this one should set, for example,

$$u_1=u_0,\ u_2=\bar{u}_0+2iK',\ B_2=B_1=0,\ A_2=-\bar{A}_1.$$

Then the system of equations (22)-(23) gives the following expressions for the coefficients b, d, f and h:

$$\begin{aligned}
b&=-W_1(u_0)\bar{e}_u-\overline{W_1(u_0)}e_u,\\
d&=i(\overline{W_2(u_0)}e_u-W_2(u_0)\bar{e}_u),\\
f&=e_u\bar{e}_u(W_3(u_0)+\overline{W_3(u_0)}),\\
h&=i(W_1(u_0)\overline{W_2(u_0)}+\overline{W_1(u_0)}W_2(u_0)-e_u\bar{e}_u(W_1(u_0)W_2(u_0)+W_1(u_0)W_2(u_0)))(W_3(u_0)-\overline{W_3(u_0)})^{-1},
\end{aligned} \tag{25}$$

where $e_u=\exp\{-ixW_3(u_0)+2itW_1(u_0)W_2(u_0)-\rho-i\varphi\}$. Substituting (25) into (24), we get an expression for a breather with nonzero momentum.

The general case $B_1\neq 0, B_2\neq 0$ corresponds to a solution which describes the interaction of two breathers.

5. Thus, we have considered all elementary excitations, and a certain classification of them appeared for us. We compare it with the classification given in [1] with respect to the zeros of the analytic function $a(u)$ (cf. [1]) situated in the rectangle $\Pi=\{u: 0<\operatorname{Im}u<2K', 0\leq\operatorname{Re}u<2K\}$. To a soliton corresponds a zero u_0 of the function $a(u)$ lying on the line $\operatorname{Im}u_0=K'$, and a breather is characterized by a pair of zeros

$$u_{1,2}=u_b\pm i\theta+iK',\ 0<\theta<K',\ 0\leq u_b<2K,$$

symmetrically situated with respect to the line $\operatorname{Im}u=K'$.

In all cases we have gotten explicit expressions for the Ψ-function, so naturally there is no difficulty in calculating the coefficient $a(u)$. For the one-soliton solution (16) we get the following expression:

$$a(u)=-\sqrt{\frac{(W_3(u)-W_3(u_0))(W_2(u)W_1(u_0)-W_1(u)W_2(u_0))}{(W_3(u)+W_3(u_0))(W_2(u)W_1(u_0)+W_1(u)W_2(u_0))}}.$$

This function has a unique simple zero at the point $u_0\in\Pi$. To the solutions (17-20) there correspond two zeros: $u_1=u_0\in\Pi$ and $u_2\in\{-u_2\}\cap\Pi$, but in the case of two-soliton solutions (17, 18) they are situated on the line $\operatorname{Im}u=K'$, and for breathers (19, 20) they are symmetric with respect to this line. The solution described by (24, 25) is a breather with nonzero momentum according to the classification of E. K. Sklyanin, since the corresponding coefficient $a(u)$ has zeros at the points $u_0, \bar{u}_0+2iK'\in\Pi$.

We note finally that knowing the zeros of the coefficient $a(u)$, corresponding to the solutions (17-20), (24), (25), we can easily calculate the momentum and energy of these solutions by the formulas in [1].

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HAMILTONIAN STRUCTURE OF POLYNOMIAL BUNDLES

UDC 519.4

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The Hamiltonian structure of nonlinear evolution equations for which the potentials of the Zakharov–Shabat system depend polynomially on a spectral parameter is investigated on the basis of a general group theoretic scheme. The orbits of the corresponding coadjoint action are calculated. Formulas for the generating functions of the densities and flows of conservation laws and $\mathfrak{M}$-operators are derived.

By a polynomial bundle we mean a linear operator $\mathfrak{L}=\partial/\partial x-\mathfrak{U}(x,\lambda)$ in which the matrix potential $\mathfrak{U}(x,\lambda)$ is a polynomial in λ with coefficients depending on x: $\mathfrak{U}(x,\lambda)=\sum_{0\leq k\leq N}u_k(x)\lambda^k$. Some evolution equations which are in involution with one another are connected with the operator $\mathfrak{L}$. These equations can be written down as the commutativity con-

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