Robust Feedback Controllers for a Backward Facing Step Flow

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Abstract. The paper investigates the control of flow over a backward facing step for a Reynolds number of 3000. The top of the channel is considered as a slip wall and in the lateral direction a periodic behavior is assumed.
With the aim of drag reduction or a reduction of reattachment length in the wake of the step, acoustic manipulations of the boundary layer in front of the step were performed by experiments and numerical simulations ([1] and [4]). A good agreement was found between the experimental and numerical results, especially in the case of the reattachment length, also related to the mean velocity field and the rms-profiles. The numerical investigations are done as direct numerical simulations and large eddy simulations.
The influence of manipulation parameters on the recirculation length X_r was identified by a series of large eddy simulations of a transitional backward facing step flow.
Because of the huge amount of computational work for the solution of an optimization problem for the drag minimization (the non stationary Navier-Stokes equation and the adjoint problem are to be solved several times...) we are looking for computationally "cheap" control strategies. Our interest is focussed on the relation of the amplitude A of the acoustic manipulation signal and the recirculation length X_r.

1 Modelling of the flow problem

We consider a backward facing step channel and we investigate the flow for a Reynolds number of 3000 built with the velocity $U_\infty = 2,27\,\frac{m}{s}$ of the block inflow profile and the step height $H = 20\,mm$ (see fig. 1) for an air flow. The top of the channel is considered as a slip wall and in the lateral direction a periodic behavior is assumed.

To describe the flow we get from the nondimensional Navier-Stokes equation the equation system for the filtered quantities

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} + \nabla \cdot \overline{\mathbf{u}}\,\overline{\mathbf{u}} = -\nabla\psi + \nabla \cdot 2\nu\overline{S} \tag{1}$$

$$\nabla \cdot \overline{\mathbf{u}} = 0 \tag{2}$$

where ν is the total eddy-viscosity

$$\nu = \frac{1}{Re} + \nu_t,$$

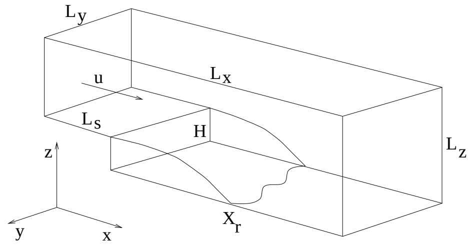

Fig. 1. channel situation

$\mathbf{u} = (u_1, u_2, u_3)$ is the velocity vector, ψ is the "pseudo"-pressure with

$$\psi = \overline{p} + \frac{1}{3}\tau_{ii}, \qquad \tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j,$$

and for the strain rate tensor $\overline{S}$ we have

$$\overline{S} = (\overline{S}_{ij}), \quad \overline{S}_{ij} = \frac{1}{2}(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}).$$

In the case of a direct numerical simulation we have $\nu_t = 0$ and there is no effect of filtering, and the equations (1) and (2) are the classical Navier-Stokes equations with $\overline{u} = u$ and $\overline{p} = p$ for an incompressible fluid.
If we don't resolve all small structures by the spatial discretization ν_t is the eddy-viscosity of a subgrid-scale model. We use a subgrid-scale model of Germano-type following Akselvoll and Moin [3]. For ν_t we have

$$\nu_t = C\overline{\Delta}^2 |\overline{S}|, \qquad |\overline{S}| = \sqrt{2\overline{S}_{ij}\overline{S}_{ij}}. \tag{3}$$

For the Germano-type subgrid scale model we need two different filters to handle the equation of motion, the so-called grid filter $\overline{G}$ and the test filter $\widehat{G}$, with the filter $\widehat{\overline{\Delta}}$ of the test filter, assumed to be larger than that of the grid-filter. The quantity $C\overline{\Delta}^2$ we set

$$C\overline{\Delta}^2 = -\frac{1}{2}\frac{< L_{ij}M_{ij} >_y}{< M_{kl}M_{kl} >_y}, \tag{4}$$

where,

$$L_{ij} = \widehat{\overline{u}_i \overline{u}_j} - \widehat{\overline{u}}_i \widehat{\overline{u}}_j, \qquad M_{ij} = (\widehat{\overline{\Delta}}/\overline{\Delta})^2 |\widehat{\overline{S}}|\widehat{\overline{S}}_{ij} - \widehat{|\overline{S}|\overline{S}_{ij}}.$$

$< \quad >_y$ indicates an average taken over the homogeneous spanwise direction. The result of the subgrid-scale modelling is concentrated in the variable eddy-viscosity. The equation of motion is of the same type as the Navier-Stokes equation.
With suitable boundary conditions which will be specified in the following section and appropriate initial conditions we have a mathematical model for the transitional flow over a backward facing step.

2 The geometry of the backward facing step and the manipulation principle

On the inflow plane and for solid walls Dirichlet boundary conditions are given (inflow block profile, no slip conditions). In the lateral direction we assume a periodic behavior and at the top boundary a slip one is used. On the outlet zero gradient conditions are used. With appropriate initial conditions now we have a mathematical model for the transitional flow over a backward facing step. The figure 2 shows the kind of manipulation, which results in Dirichlet-boundary conditions.

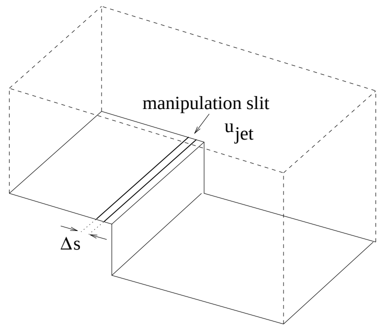

Fig. 2. Acoustic manipulation via a spanwise manipulation slit, $\Delta s \approx 0.05\, H$

For the streamwise length L_x, the spanwise width L_y and the vertical height L_z we set $(L_x, L_y, L_z) = (22H, 6H, 12H)$. The choice of the vertical height is based on experiences concerning the dependency of the reattachment length on L_z. Only beyond $11H$ the dependency of the reattachment length can be neglected. The inlet section of the step has length $L_s = 5H$. The inflow profile was assumed as a block profile with the velocity $U_\infty = const$.
Non-uniform grid spacings for the streamwise and vertical directions are used. In the x- and the z-directions we consider a refined grid around the step. In z-direction fine spacings are used near the channel bottom. In the spanwise direction uniform grid spacings are used.
Because of the absence of detailed information about the blowing and suction during the acoustic control it was simulated by a sine function of the form $U_{jet} = A\ sin(2\pi\, f\, t)$ $(U_1 = U_2 = U_{jet})$. Former calculations [4] with amplitudes of $A = 0.1\, U_\infty$, $A = 0.01\, U_\infty$ and $A = 0.001\, U_\infty$ using a fixed frequency of $f = 50\, Hz$ showed a good agreement with the experimental results [1] using different loudnesses of the loudspeakers. With the used physical dimensions the dimensionless frequency has the value $F = \frac{fH}{U_\infty} = 0.45$.

3 Identification

Because of the huge amount of computational work for the solution of an optimization problem for the drag minimization (the non stationary Navier-Stokes equation and the adjoint problem are to be solved several times...) we are looking for computationally "cheap" control strategies. Our interest is focussed on the relation between the amplitude A of the acoustic signal and the recirculation length X_r.
The recirculation length X_r is determind as the last intersection with the x-axis of the graph of wall shear stresses (x-direction) averaged in the lateral direction. Figure 3 shows a typical distribution of the derivative of $\frac{\partial U_1}{\partial z}$ at the wall, which ist proportional to the wall shear stress τ_w.

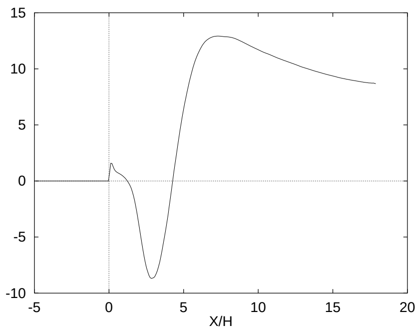

Fig. 3. $\frac{\partial U_1}{\partial z}$ at the channel bottom averaged in the lateral direction

In the result of the large eddy simulations for several parameters constellations different black box models have been identified. In the most simple case a first order model of the form

$$T\,X_r'(t) + X_r(t) = K\,A(t) \tag{5}$$

could be used to describe the dependence of the reattachment length on the amplitude of the harmonic excitation.
Figure 4 shows the time response of the recirculation length to a step of the manipulation amplitude based on the LES and the black box model (from $A = 0$, non manipulated flow, to $A = 0.01U_\infty$, manipulated case). A second order approximation gives only a slight improvement of the fit. However, if this approximation is repeated for other Re-numbers different parameters T and K are identified. Therefore a family of models was considered to design a controller.

The first black box model was used as a basis for a robust controller design. Different control schemes were tested. Only the very simple P-controller is shown here. It's equation is of the form

$$A(t) = K_p[W(t) - X_r(t)] \quad , \tag{6}$$

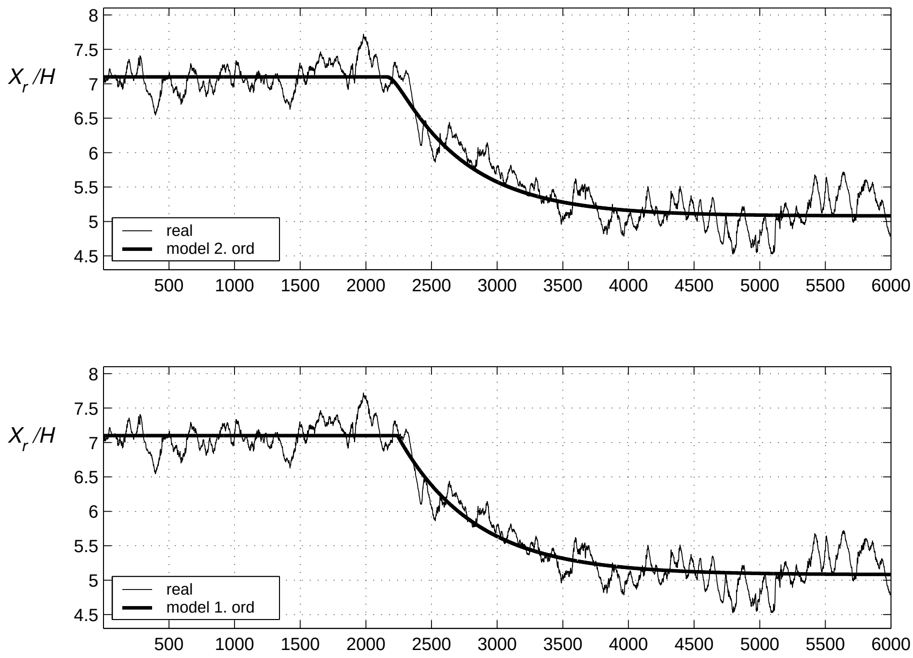

Fig. 4. time response to an amplitude step

where W stands for the setpoint of the recirculation length and k_p is the gain factor of the controller. For a stable closed loop it is necessary to work with the factor $K_p = -0.01\frac{U_\infty}{H}$. If the same controller is to be used for other larger Reynolds numbers, the gain has to be reduced to maintain robust stability. To guarantee zero steady state offset for a constant setpoint a PI-controller can be used which is described by

$$E'(t)K_c\tau_c + E(t)k_c = A'(t)\tau_c \quad , \tag{7}$$

with

$$E(t) = W(t) - X_r(t),$$

where τ_c, K_c are constant system parameters ([2]). Figure 5 shows the time behavior of the recirculation length and the manipulation amplitude during the work of the P-controller. A recirculation setpoint length $W = 5\,H$ was prescribed.

The study of more robust controllers like Smith-predictors to increase performance is under consideration.

4 Further investigations

As mentioned above the huge amount of cpu-time for the solution of an optimisation problem with respect to functionals measuring the reattachment length

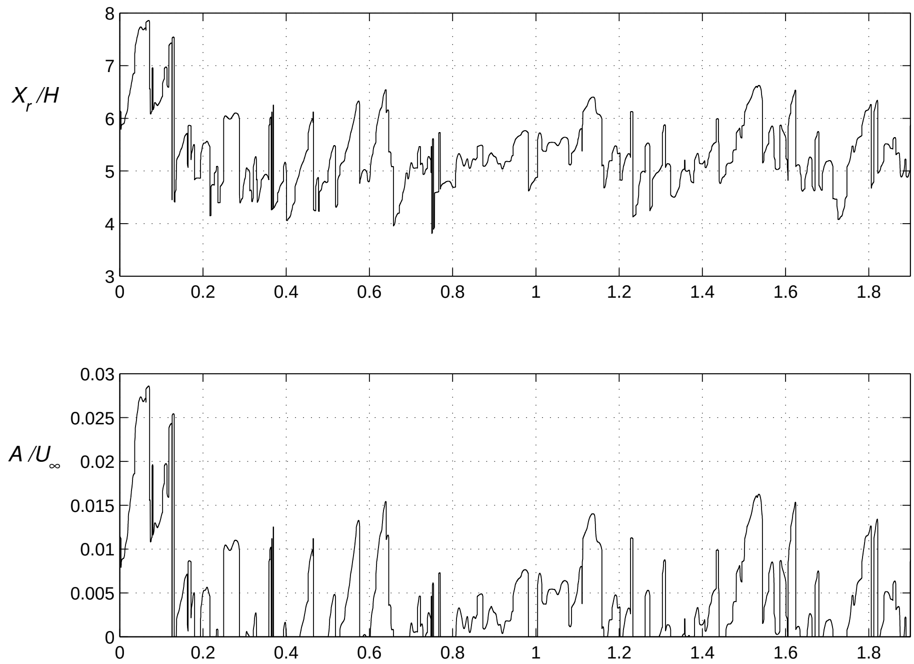

Fig. 5. numerical simulation of the P-controller, $X_r(t)$ and $A(t)$

is the reason for the discussed identification methods to construct robust controllers. But the computational work for the identification is still very large and we think about the development of our flow by a finite number of modes in the result of a proper orthogonal decomposition using snapshots of an unsteady numerical simulation of the manipulated backward facing step flow.

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