Modelling unbalanced pedestrian flow on a two-dimensional grid

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Objectives

- To some extent, pedestrian flow can be considered as a compressible fluid.
- However, since any real pedestrian would claim a minimum physical space, pedestrian flow becomes incompressible once the density reaches a threshold value.
- The size of this minimum physical space may even vary under circumstances of different social contexts.
- Thus, we discuss the possibility of describing this phenomenon on a two-dimensional grid. On this grid, the notion of ”virtual grid cell” is applied to describe the aforesaid minimum space.
- The basic idea of our model is applicable for the general grid-based methods of pedestrian flow simulation.

Basic idea of pedestrian flow modeling by Cellular Atomata

- The CA models are generally categorized to be microscopic, that is, the pedestrians (called *particle*s) are investigated individually.
- It is obvious that at every position represented by the cells of the automaton geometrically, there can be only two possibilities: either, exactly one particle is currently located at this position, or no particle is at this position.
- The special cell state is always well-defined by the certain appropriate transition rules.
- ”Compressible” effects are considered to model real life situations.

Compressibility

We consider a simulation system of pedestrian dynamics in which the sink (flow-out) is not equal to the source (flow-in). With the unbalancing source and sink, the particle density varies. The particle density has a twofold influence in the simulation: it is related to the inverse of the minimum space which the particles claim and at the same time, it decides the local particle velocities. We investigate the pedestrian flow in this context. To make our model independent of other factors, we neglect the more specific interactions among the particles, the particles are given a destination and they form a flow on a global basis accordingly.

Grid construction

We consider two extreme cases in which a particle (a real pedestrian) reserves a space of 0.9×0.9 and 0.3×0.3 respectively.

This enables us to construct the underlying grid with a cell size of 0.3×0.3 for the simulation; and a planar space of 0.9×0.9 refers to a set of 3×3 grid cells. We consider two states of the eight surrounding grid cells (with a particle's current position as the origin): *activated* (abbreviated:) or *deactivated* (abbreviated:).

Grid construction

Figure 1: Activating and deactivating grid cells.

Figure 2: Associating grid cells with a probability of being deactivated.

Activating-deactivating of cells

We may describe the grid cell state of or by assigning a probability of deactivation p, see Figure **??**.This probability p is further dependent on an index variable $\imath$: $\imath = 1$ denotes the normal case mentioned above in which a particle claims nine grid cells whereas $\imath = 0$ denotes the other case in which only exactly one cell is requested.

Particle density

A straight way to express the particle density is the mass of the particles per volume. After adapting it to the two-dimensional grid and re-scaling both the particle volume and the grid size, this becomes

$$\rho = \frac{\text{number of the particles}}{\text{number of the grid cells}} \, .$$

In this way, the density is computed in two dimensions. Similarly, this density can be calculated on a local basis. Since the current model focuses on the simplest situation in which the particles are given a fixed destination, the position change of the particles in the second dimension (with the first being the flow direction) should be less interesting than in the first dimension. Thus, it suffices us to consider the density in one dimension. In the sequel, density will be computed columnwise on a two-dimensional grid.

Flow-in densities

In our model of simulation, particles will be produced on a pre-configured basis in the leftmost column of the grid. We call these “flow-in”. (Similarly, particles leaving the rightmost column of the grid will be called “flow-out”.) In Figure **??**, the flow-in rates (in comparison to the size of column, the number of grid cells in one column) of the particles can be considered as the particle densities in the first column of the two-dimensional grid. In the notation of *ChenBaerwolffSchwandt09b, the columns represent the second dimension of the particle flow, the deviation from the local flow direction.

Figure 3: Different flow-in densities: 0, 1, $\frac{1}{2}$ and $\frac{1}{3}$. The particles flow from the left to the right.

Model of activation by probabilities

In Figure **??**, the particle density in the first column of the grid is 0, 1, $\frac{1}{2}$ and $\frac{1}{3}$ respectively. Among these, 1 and $\frac{1}{2}$ present two critical cases. 1 implies that a particle takes exactly one grid cell, and refers therefore to the threshold of the particles' compressibility and indicates $\imath = 0$. $\frac{1}{2}$ implies that a nontrivial (not on the boundary etc.) particle is positioned in a grid cell with two further empty grid cells on both sides. Therefore, for a density ρ, the indicating index $\imath$ for the deactivation probability p can be defined in the following way,

$$\imath : [0,1] \to [0,1],$$

$$\imath = \begin{cases} 1, & \text{if } 0 \leq \rho \leq \frac{1}{2}, \\ 2(1-\rho), & \text{if } \frac{1}{2} \leq \rho \leq 1, \\ 0, & \text{if } \rho = 1. \end{cases}$$

Flow-in vs. flow-out

To realize the situation in which the sink (flow-out) is not equal to the source (flow-in) we introduce the following ratio

$$\alpha = \frac{\text{number of grid cells in the first column}}{\text{number of grid cells in the last column}} \tag{1}$$

Without loss of generality, we assume $\alpha \geq 1$ (the case $\alpha \leq 1$ can be interpreted in full analogy) $\imath$ and $\imath$ denote the indicating index variable for deactivation probability for flow-in and flow-out respectively.

For $\alpha\rho_{>}1$, particle congestion would be unavoidable, because

$$\alpha\rho_{=}\frac{\text{flow-in capacity}}{\text{flow-out capacity}} \cdot \rho,$$

and therefore,

$$\text{flow-in capacity } \cdot \rho_{>}\text{flow-out capacity} \cdot 1.$$

To achieve a balanced flow, ρ must be larger than 1, which is not possible. In the test cases described later, we will see this confirmed.

Velocity and step calculation on the grid (1D)

In the research of vehicular traffic flow, the correlation of the density of the vehicles and their average velocities had been discovered at an early stage by Greenshields, Kuehne (2008) and adapted in models of Lighthill/Whitham. This phenomenon can be described by

$$v(\rho) = v_0 \left(1 - \frac{\rho}{\rho_{\max}}\right).$$

This yields

$$\frac{\partial v}{\partial \rho} = -\frac{v_0}{\rho_{\max}},$$

and gives us an approximation of the velocity change in particular in the vicinity of $\rho = 0$. Overall, the velocity v tends to take the undisturbed value v_0 when the density ρ is low, and for high densities, the velocity v decreases to zero due to congestions.

Velocity and step calculation on the grid (2D)

On a two-dimensional grid, $\rho_{\max}$ can be simply set to 1, since in each of the grid cell of the underlying geometry there can be at most one particle present. For the individual particles, since its moving speed is dependent of the environmental settings, ρ should be evaluated locally.

Application setting

We apply the basic configuration of *ChenBaerwolffSchwandt09a for $t = 1$ as the time length of each simulation step, $v_0 = 1.5\ ^{-1}$ as the undisturbed average velocity of the pedestrians. Given the grid cell of size of 0.3 in one dimension, a normal, undisturbed step would correspond to five cells on the grid. We refer to *ChenBaerwolffSchwandt09a for the step calculation for multiple cells using J. E. Bresenham's algorithm. The step computed in this way stands for the shortest way from one position to another on the two-dimensional grid. To make the model independent, we introduce no further step calculation schemes. In other words, the particles merely "flow" from one end (the source) to the other (the sink) of the simulation system.

Notes on implementation

Figure 4: Saving all the deactivation information for a grid cell in a container holding up to eight items. Each item contains a pointer to the position of the affecting particle and the relevant value of deactivation probability.

Notes on implementation

For a grid cell under multiple influence of deactivation, there are two possible interpretations of the deactivation probability. Suppose a grid cell is to be deactivated by its neighbour N_1 with a probability p_1 and a second neighbour N_2 with p_2. When we take these two events as independent, the probability for a deactivation for this grid cell would be

$$1 - (1 - p_1)(1 - p_2) = p_1 + p_2 - p_1 p_2,$$

since it must not be deactivated by N_1 and at the same time, not by N_2, either. On the other hand, due to the heuristic nature of the construction of p the alternative way

$$\max\left(p_1, p_2\right)$$

offers an acceptable approximation. Referring to Figure **??**, this means that the largest value among all items plays a key role for the grid cell investigated. Since the relative positions of other particles may change in every simulation step, this deactivation probability evolves in the simulation.

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