Modeling of pedestrian flows using hybrid models of Euler equations and dynamical systems

Günter Bärwolff, Thomas Slawig and Hartmut Schwandt

Technische Universität Berlin, Institut für Mathematik

Abstract. In the last years various systems have been developed for controlling, planning and predicting the traffic of persons and vehicles, in particular under security aspects. Going beyond pure counting and statistical models, approaches were found to be very adequate and accurate which are based on well-known concepts originally developed in very different research areas, namely continuum mechanics and computer science. In the present paper, we outline a continuum mechanical approach for the description of pedestrain flow.

Keywords: Navier-Stokes equation; Euler equation; traffic simulation; social behavior model
PACS: 45.70.Vn, 47.10.ab, 47.10.ad, 47.11.-j, 47.11.Bc, 47.57.G

INTRODUCTION

In this section we present an extension of a two-dimensional macroscopic pedestrian flow model based on the compressible Euler equation by introducing local, in a certain sense microscopic effects.

In macroscopic traffic/pedestrian flow models based on fluid dynamic equations, all quantities are usually given and computed in Eulerian coordinates. Thus it is (only!) possible to state that a person at time t at point $\vec{x}$ has, e.g. velocity $\vec{v}(\vec{x},t)$ etc. For instance, it is conceptually impossible to endow a group of pedestrians with a constant predefined target direction throughout the whole simulation, whereever this group will be at any time t. This disadvantage gives rise to introduce effects or forces that are considered in Lagrangian coordinates, i.e. which depend on the spatial starting point $\vec{x}_0 = \vec{x}(t_0)$ of the simulation time interval.

These extensions can be used to model and simulate different groups of pedestrians with different behavior throughout the model time. Thus it can be used to incorporate target directions, ”pedestrian jams” due to mixing of crowds with different targets, and temporal changes of pedestrian groups.

In the following, we present the underlying macroscopic model of pedestrian flow, describe the idea of introducing extensions based on the Lagrangian coordinates and show how this can be realised with a modern Finite Element software.

A HYBRID MODEL OF PEDESTRIAN MOVEMENTS BY EXTENSION OF A MACROSCOPIC FLUID DYNAMIC MODEL

Traffic and pedestrian flow modeling can be divided into two types, the microscopic and the macroscopic approach. In the former, all vehicles/persons are treated individually and equations for their velocity and interactions are posed. The main types of this model class are cellular automata and multi-body systems. By contrast, in the macroscopic approach all vehicles/persons are considered as a continuum. In this case, a mathematical model for density, velocity and other relevant quantities is formulated.

In a realistic model of pedestrian movements, forces between individual persons have to be taken into account: For example, if the density of persons is very high and thus the distance between them becomes small, a repelling force between the individual persons arises. This motivates the use of a gas kinetic and particle-oriented approach based on the Boltzmann equation. Solving this equation directly leads again to microscopic models.

Introducing mean values in the Boltzmann equation is one way to derive the well-known equations of fluid mechanics, and this is a motivation for using the latter for vehicle and pedestrian movements. On the other hand, one may directly make the continuum assumption for the whole bulk of vehicles/persons. This directly leads to models that have strong relation to compressible fluid dynamic equations, as for example the Burgers (in one space dimension) or Euler equations (in two dimensions). In pedestrian movements, two dimensional effects are crucial. This marks the main difference to (vehicle) traffic models, which mainly consists of several one-dimensional models, one for each lane of vehicle traffic.

THE EULER EQUATIONS FOR TWO-DIMENSIONAL PEDESTRIAN FLOW

In 2-D compressible flow, the relevant quantities are density ρ, velocity vector $\vec{v} = (u, v)$, and energy. For special gases, the energy can be eliminated using the temperature θ. Following Helbing [1], the equations are formulated for mean values (denoted by $\langle\cdot\rangle$) of velocity and density. Here, the analogue of the temperature in pedestrian flow is the variance $\theta := \langle(v_{||} - \langle v_{||}\rangle)^2\rangle$ of the velocity $v_{||}$ parallel to the intended velocity direction. The basic governing equations for pedestrian flow in this model then read

$$\left.\begin{array}{rcl} \rho_t + \nabla\cdot(\rho\vec{v}) & = & 0 \\ \vec{v}_t + \vec{v}\cdot\nabla\vec{v} + \frac{1}{\rho}\nabla(\rho\theta) & = & F \\ \theta_t + \nabla\cdot(\theta\vec{v}) & = & 0 \end{array}\right\} \quad (1)$$

in $\Omega \subset \mathbb{R}^2$. This nonlinear set of equations has the form of the Euler equations with (formally) an adiabatic index $\gamma = 2$.

The form of external and also internal forces F is subject to further modeling concepts, see again [1]. Also, an additional diffusivity term may be introduced. Appropriate boundary and initial conditions have to be added.

NUMERICAL METHODS

The PDE system presented above (without diffusion term) is of hyperbolic type. Thus steep gradients, shocks and discontinuities may occur. When applying finite elements, the numerical treatment requires sophisticated stabilisation schemes to obtain a solution. Basically two terms are added to weak form of the equations: A streamline upwinding part and a small and successively reduced anisotropic diffusion term, see for example [2]. Our computations were performed in the modeling and simulation environment COMSOL MULTIPHYSICS™[3].

Special feature of the hybrid modeling concept: Realisation of terms in Lagrangian coordinates

The main additional feature that is missing in macroscopic models coming from fluid dynamics, as the Euler equations, is that all quantities $\rho, \vec{v}, \theta$ are given as functions of time t and the current location $\vec{x}$ of the fluid particle (i.e. pedestrian). It is thus easy to introduce forces at fixed locations (for example an ”attraction” as a shop window that attracts everybody who comes near to it, maybe with a certain probability). On the other hand, it is impossible to endow one or even several pedestrian group(s) with a persistent intention (e.g. for a certain target direction), no matter in which positions $\vec{x}$ the members of the group are located at time t. For this purpose we extend the model by introducing terms in Lagrangian coordinates. This means that the initial positions $\vec{x}_0 := \vec{x}(t_0)$ are propagated in time during the simulation. The current positions $\vec{x} = (\vec{x}_0, t)$ satisfy the initial value problem

$$\frac{d}{dt}\vec{x}(t) \quad = \quad \vec{v}(\vec{x}, t), \quad \vec{x}(t_0) \; = \; \vec{x}_0. \quad (2)$$

This ODE system is solved together with (1) in the following way:

1. Choose one or more pedestrian groups G^j, $j = 1, \ldots, j_{max}$, by defining their discrete initial domains $\Omega_0^j = \{\vec{x}_0^{ji} : i = 1, \ldots N_j\}$ and forces $\vec{f}_k^{ji}$, where k refers to a discrete time $t = t_k$.
   Such a force may describe an intended direction of a group member with index i which at time $t = t_k$ is at the point $\vec{x}_k^{ji}$. The force may use the distance vector of this point to an attraction or target point $\vec{x}_{tar}$. Since we use an integral form of the equations (1) it is necessary to define the force not only pointwise, but in a neighborhood of

$\vec{x}_k^{ji}$, e.g.

$$\vec{f}_k^{ji}(\vec{x}) := \left\{ \begin{array}{cc} c^j \frac{\vec{x}_{tar}^j - \vec{x}}{\|\vec{x}_{tar}^j - \vec{x}\|}, & \|\vec{x}^{ji} - \vec{x}\| \le r^j \\ 0, & \text{elsewhere} \end{array} \right\},$$
$$r^j \in \mathbb{R}^+, c^j \in \mathbb{R}.$$

Set $k = 0$.

2. Compute a solution of (1) including appropriate boundary conditions and initial values on an interval $(t_k, t_{k+1}]$, adding $\vec{F} = \sum_{ji} \vec{f}_k^{ji}(\vec{x})$ as force on the right-hand side. In this step, it is possible – and maybe necessary due to stabilisation reasons – to compute the solution on intermediate times $t_k = t_{k_0} < t_{k_1} < \ldots < t_{k_{n_k}} = t_{k+1}$.
3. Propagate the trajectories of the members of the pedestrian group(s) G^j, i.e. the points in the set $\{\vec{x}_{kn}^{ji} : j = 1, \ldots, j_{max}, i = 1, \ldots N_j, n = 1, \ldots, n_k\}$, by solving (2) on the discrete grid $t_k = t_{k_0} < t_{k_1} < \ldots < t_{k_{n_k}} = t_{k+1}$ using $\vec{v}(\vec{x}_{kn}^{ji}, t_{kn}), n = 1, \ldots, n_k$.
4. Increment $k \longrightarrow k+1$ and proceed with step 2.

Numerical Example

As a simple example we consider the movement of two pedestrian groups with opposite target directions.

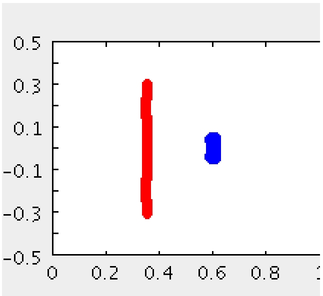

FIGURE 1. Start situation

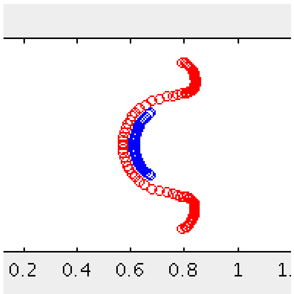

FIGURE 2. Intermediate time

Fig. 1 - Fig. 3 show the realised pedestrian movement by the mathematical model. The blue (dark) pedestrian group starting on the right is attracted by the left boundary, whereas the red (light) group starting from the left is moving to the left. From left to right: Initial time, meeting of the two groups, after passing. The top and bottom boundary indicate walls.

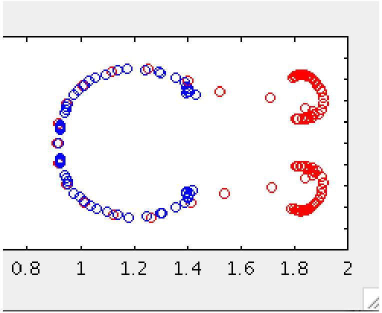

FIGURE 3. After passing

OPTIMISATION PROBLEMS

Optimisation goals can be, for example, minimising the time to evacuate buildings or computing the maximal flow rate through a narrow channel. As a state variable we consider the density of pedestrians which is described by a discrete or continuous model resulting in a partial differential equation. In this, the shape of a channel or the size of doors serve as control variables.

Based on experiences in shape optimisation problems of fluid dynamics with the adjoint optimisation calculus an optimisation can be achieved. To evaluate gradients of Lagrange functionals, we use both the automatic differentiation tools (AD-tool, for example ADIfor) and the numerical solution of the Karush-Kuhn-Tucker system (KKT system).

CONCLUSION

Next to models based on the microscopic (cellular automata) approach, the macroscopic (continuous) approach yields very flexible tools to describe, analyse, predict and control many, in their complexity varying situations. In the present paper we have have presented the hybrid macroscopic model based on the mass and impulse balance (s. also [4]). The numerical examples gave plausible results, which will be compared to results of microscopic models in the next future.

ACKNOWLEDGMENTS

This research project is supported by the Berlin Government and by the European Union (PROFIT program). Further we would like to thank our colleague Prof. K. Nagel from the Technische Universität Berlin for helpful comments.

REFERENCES

1. Dirk Helbing: *A fluid dynamic model for the movement of pedestrians*, Complex Systems, 6:391–415, 1992.
2. Olivier Pironneau: *Finite Element Methods for Fluids*. J. Wiley & Sons/Masson, 1989.
3. Comsol AB: *http://www.comsol.com*.
4. G. Bärwolff, M.-J. Chen, H. Schwandt and T. Slawig: *Simulation of Pedestrian Flows for Traffic Control Systems*, Proc. of the Sixth International Conference on Information ans Management Sciences, July 1-6, 2007 Lhasa/Tibet, in Series of Information & Management Sciences of the Cal Poly State University 2007.