Numerical Simulation of a Backward Facing Step Flow with Robust Feedback Controlled Boundary Layer Manipulations

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Abstract. With the aim of the drag reduction or a decreasing reattachment length in the wake of the step, acoustic manipulations of the boundary layer in front of the step were performed by experiments and numerical simulations ([1], [3] and [6]). The numerical investigations were done as direct numerical simulations and large eddy simulations.
Optimal control strategies are not realistic yet. Therefore robust control strategies are identified using the unsteady simulation results for various control parameter sets.

1 Mathematical model

We consider a backward facing step channel and we investigate the flow for a Reynolds number of 2500 up to 3500 built with the velocity u_0 of the block inflow profile and the step height H (see fig. 1). The top of the channel is considered as a slip wall and in the lateral direction a periodic behavior is assumed.

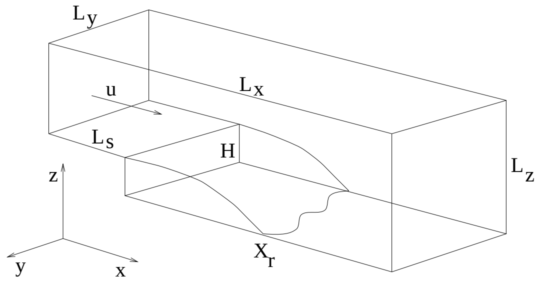

Fig. 1. channel situation

To describe the flow we get from the nondimensional Navier-Stokes equation the equation system for the filtered quantities

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} + \nabla \cdot \overline{\mathbf{u}}\,\overline{\mathbf{u}} = -\nabla \psi + \nabla \cdot 2\nu \overline{S} \tag{1}$$

$$\nabla \cdot \overline{\mathbf{u}} = 0 \tag{2}$$

where ν is the total eddy-viscosity

$$\nu = \frac{1}{Re} + \nu_t,$$

$\mathbf{u} = (u_1, u_2, u_3)$ is the velocity vector, ψ is the "pseudo"-pressure with

$$\psi = \overline{p} + \frac{1}{3}\tau_{ii}, \qquad \tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j,$$

and for the strain rate tensor $\overline{S}$ we have

$$\overline{S} = (\overline{S}_{ij}), \quad \overline{S}_{ij} = \frac{1}{2}(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}).$$

In the case of a direct numerical simulation we have $\nu_t = 0$ and there is no effect of filtering, and the equations (1) and (2) are the classical Navier-Stokes equations with $\overline{u} = u$ and $\overline{p} = p$ for an incompressible fluid.
If we don't resolve all small structures by the spatial discretization ν_t is the eddy-viscosity of a subgrid-scale model. We use a subgrid-scale model of Germano-type following Akselvoll/Moin [4]. For ν_t we have

$$\nu_t = C\overline{\Delta}^2 |\overline{S}|, \qquad |\overline{S}| = \sqrt{2\overline{S}_{ij}\overline{S}_{ij}}. \tag{3}$$

For the Germano-type subgrid scale model we need two different filters to handle the equation of motion, the so-called grid filter $\overline{G}$ and the test filter $\widehat{G}$, with the filter with $\widehat{\overline{\Delta}}$ of the test filter, assumed to be larger than that of the grid-filter. The quantity $C\overline{\Delta}^2$ we set

$$C\overline{\Delta}^2 = -\frac{1}{2}\frac{< L_{ij}M_{ij} >_y}{< M_{kl}M_{kl} >_y}, \tag{4}$$

where,

$$L_{ij} = \widehat{\overline{u}_i\overline{u}_j} - \widehat{\overline{u}}_i\widehat{\overline{u}}_j, \qquad M_{ij} = (\widehat{\overline{\Delta}}/\overline{\Delta})^2 |\widehat{\overline{S}}|\widehat{\overline{S}}_{ij} - \widehat{|\overline{S}|\overline{S}_{ij}}.$$

$< \ >_y$ indicates an average taken over the homogeneous spanwise direction. The result of the subgrid-scale modelling is concentrated in the variable eddy-viscosity. The equation of motion is of the same type than the Navier-Stokes equation.
With the above-discussed boundary conditions which will be specified in the following section and appropriate initial conditions now we have a mathematical model for the transitional flow over a backward facing step.

2 Fluid physical task

The figure 2 shows the used manipulation, which results in Dirichlet-boundary conditions.

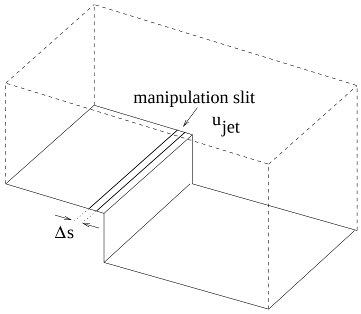

Fig. 2. Acoustic manipulation via a spanwise manipulation slit, $\Delta s \approx 0.05\,H$

For the streamwise length L_x, the spanwise width L_y and the vertical height L_z we set $(L_x, L_y, L_z) = (22H, 6H, 12H)$. The choice of the vertical height is based on experiences concerning the dependency of the reattachment length on L_z. Only beyond $11H$ the dependency of the reattachment length can be neglected. The inlet section or the step has the length $L_s = 5H$. The inflow profile was assumed as a block profile with the velocity $u_{inflow} = const.$.
Non-uniform grid spacings for the streamwise and vertical directions are used. In the x- and the z-directions we consider a refined grid around the step. In z-direction fine spacings are used near the channel bottom. In the spanewise direction uniform grid spacings are used.
Because of the absence of detailed information about the blowing and suction during the acoustic control it was simulated by a sine function of the form $u_{jet} = A\ sin(2\pi\, f\, t)$ $(u_1 = u_2 = u_{jet})$ and calculations for $A = 0.02\,u_{inflow}$ and $A = 0.01\,u_{inflow}$ using a fixed frequency of $f = 50\,Hz$ were done. For the determination of a data base with respect to the robust control the Reynolds number and the amplitude A were varied.

3 Numerical solution procedure

Equations (1) and (2) may be discretized in spatial dimensions by a finite volume method on staggered grids for the velocity components u_i of the velocity vector and the pressure ψ[1].
The finite volume discretization results in a ode system for the velocity components at every grid point

$$\frac{\partial \mathbf{u}_h}{\partial t} + \nabla_h \cdot \mathbf{u}_h \mathbf{u}_h = -\nabla_h \psi_h + \nabla_h \cdot 2\nu \overline{S}_h \quad , \tag{5}$$

$$\nabla_h \cdot \mathbf{u}_h = 0, \tag{6}$$

written in vector form. The approximation is conservative and is of second order $(O(h^2))$. The time integration is done either by a leapfrog method or

[1] For a compact description we renounce the lines over u, ψ.

an Adams-Bashforth method. Thus, we have to solve in every time step the equation system

$$\frac{\mathbf{u}_h^{n+1} - \tilde{\mathbf{u}}_h}{\tau} = -\beta \nabla_h \psi_h^{n+1} \quad , \quad \nabla_h \cdot \mathbf{u}_h^{n+1} = 0 \,, \tag{7}$$

where $\tilde{\boldsymbol{u}}_h$ is a given result of an estimation by a predictor step, τ is the time step and β is a constant depending on the time integration method used (in the case of the Adams-Bashforth method $\beta = 1$, see also [2]).
The equations (7) are equivalent to the equation:

$$-\Delta_h \psi_h^{n+1} = -\frac{1}{\tau\beta} \nabla_h \cdot \tilde{\mathbf{u}}_h \quad . \tag{8}$$

$\mathbf{u}_h^{n+1}$ is then given by an explicit fill-in step following (7). Thus, $(\mathbf{u}_h^{n+1}, \psi_h^{n+1})$ can be found either iteratively or by the solution of a Poisson equation for ψ^{n+1} followed by an explicit fill-in step to get $\mathbf{u}^{n+1}$. Both possibilities are implemented in the sequential codes (see [2],[7]). In our parallel code implementation the iterative solution method to get $(\mathbf{u}_h^{n+1}, \psi_h^{n+1})$ is realized as a solver for the equation system (7). The numerical simulations are done with a finite volume method on staggered grids which is parallelized on a Cray T3D/T3E using fast Cray-specific shared memory transfer utilities or the platform independent MPI library ([5]). The method is of second order in space and time. The mass conservation per time step was realized by a pressure-velocity iteration method.
Non-uniform grid spacings for the streamwise and vertical directions are used. In the x- and the z-directions we consider a refined grid around the step. In z-direction fine spacings are used near the channel bottom. In the spanwise direction uniform grid spacings are used.
The parallelized numerical model (DNS/LES) was validated by comparisons of the numerical DNS/LES results of [6] and the experimental data of [1] and [8]. The following figures 3-6 show the result of comparison for the mean velocities and the rms-values. For these comparisons a neutral, non-manipulated backward facing step flow for the transitional Reynolds number 3000 was considered. For the direct numerical simulations about 10 million grid points are needed for the spatial discretization ($516 \times 132 \times 196$). In the case of the large eddy simulations it is possible to work with about 475 thousands of spatial grid points ($132 \times 60 \times 60$). The experimental results were produced by LDA-measurements (denoted by stars in the figures).
In addition to the named mpp systems we are also very successful using the SGI O 200 for the solution of moderate problems with less than 10^6 grid points.

4 Results

The results of the reattachment length reduction in the case of the acoustic manipulation over a slit show a good agreement with the known experimental

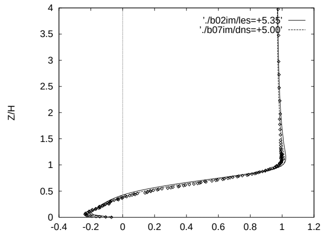

Fig. 3. u_{mean} at $x = 5H$

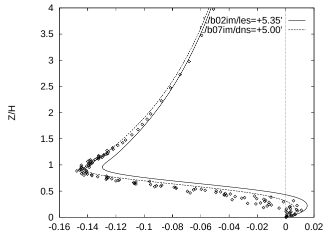

Fig. 4. w_{mean} at $x = 5H$

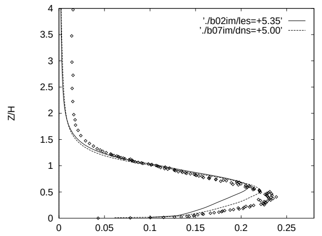

Fig. 5. u_{rms} at $x = 5H$

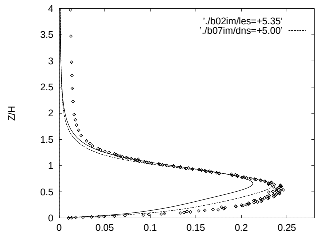

Fig. 6. w_{rms} at $x = 5H$

measurements. This is valid for both the large eddy simulation and the direct numerical simulation.
The figures 7 and 8 show the resulting wall shear stress distribution (u_τ) of a large eddy simulation with acoustic manipulation ($A = 0.01\, u_0$) compared to u_τ of the unmanipulated step flow (neutral step flow). There were no visible differences in the wall shear stress distribution of a LES and a DNS. To

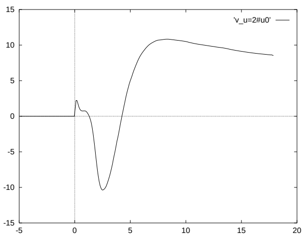

Fig. 7. u_τ of the "manipulated" step flow

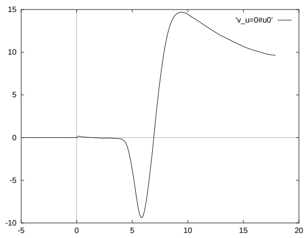

Fig. 8. u_τ of the "neutral" step flow

get an impression of the instantaneous flow situation at the channel bottom behind the step we look at the velocity and pressure fields of the large eddy simulation. The figures 9 and 10 show the streamwise velocity contours in the plane $z = 0.01\, H$ (the first inner point plane of the computational grid parallel to the bottom) took from instantaneous snapshots of the LES.
The figures 11 and 12 show the pressure contours in the plane $z = 0.01\, H$

Further results especially colour figures of pressure and vorticity contours of the investigated backward facing step flow are presented on my webpage `http//www.math.tu-berlin.de/~baerwolf`.

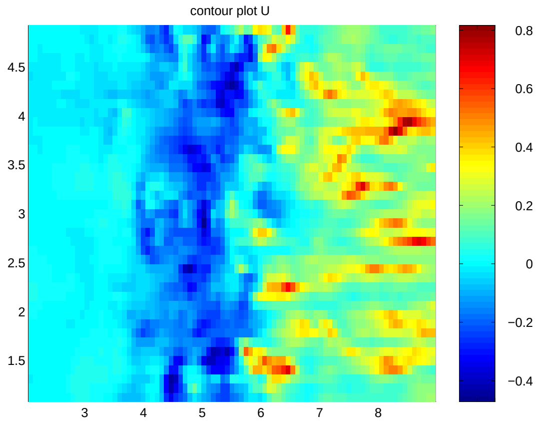

Fig. 9. U contours at $z = 0.01\,H$, neutral

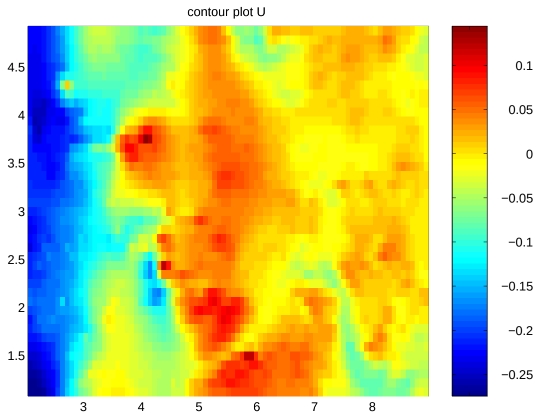

Fig. 10. U contours at $z = 0.01\,H$, manipulated

5 Control

For the solution of a genuine optimal control problem we have to find the minimum of a functional $F(A)$, which measures the quality of the result with respect to the drag reduction (decrease of the recirculation length) for example for a given amplitude of the manipulation.
To get a minimum of $F(A)$ we have to calculate the gradient of F and this means

a) the solution of the unsteady Navier-Stokes equation for a given amplitude function $A(t)$, $t \in [0, T]$, where $[0, T]$ is an interesting time interval of our problem, and
b) the solution of the adjoined problem over the reverse time interval $[T, 0]$.

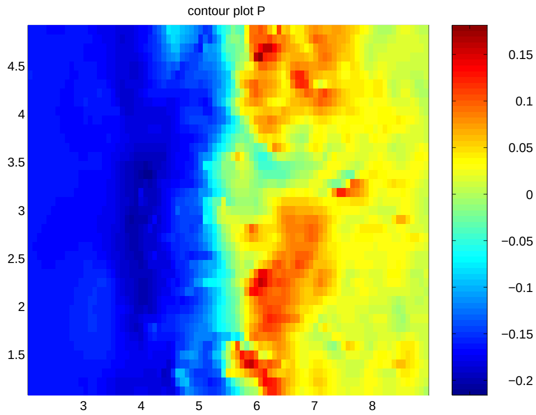

Fig. 11. P contours at $z = 0.01\,H$, neutral

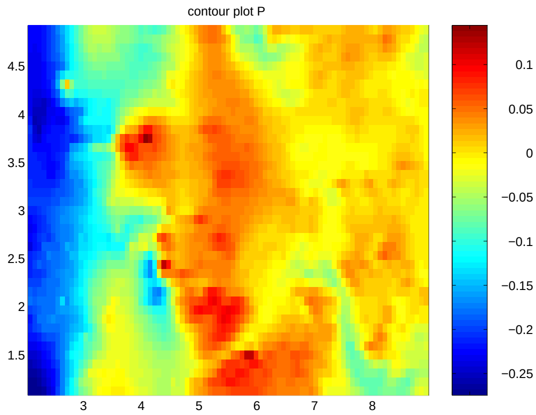

Fig. 12. P contours at $z = 0.01\,H$, manipulated

Because of the huge amount of cpu time and memory it is not possible to solve the optimal control problem in a moderate finite time.
Thats why we go another way. We do a numerical simulation using a prescribed time dependend amplitude function $A(t)$, $t \in [0, T]$, $T = 8\,sec$ shown in figure 13. The resulting time dependend recirculation length is shown in figure 14.
We do these numerical simulations for several Reynolds numbers (2500, 3000, 3500) and maximal amplitudes of $0.01u_0$ and $0.02u_0$. Thus we create a data base for the indentification of a linear equation of the type

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_0 y = b_0 u + \ldots + b_m u^{(m)} , \tag{9}$$

where $y := X_r$ stands for the recirculation length and $u := A$ stands for the amplitude A of our manipulation. We understand by the instantaneous

recirculation length the last point of interaction of the wall shear stress distribution with the $x-$axis.
The parameters a_j, $j = 0, ..., n$, b_i, $i = 0, ..., m$, are fitted in the result of the identification of our above discussed data base. The first identification results show the sufficiency of $n, m \leq 2$.
With the linear robust controller (9) its now possible to answer a measured recirculation length by an suitable amplitude.
To overcome the difficulty of measuring the recirculation length by analyzing the wall shear stress distribution we use the fact, that the recirculation lenght X_r correlates in a certain manner with the maximum of pressure fluctuations at the channel bottom in the wake region.
In the practical realization of the control strategy the pressure fuctuations will be measured by an array of microphones at the channel bottom. The controller is now implemented in our wind tunnel with the backward facing step.

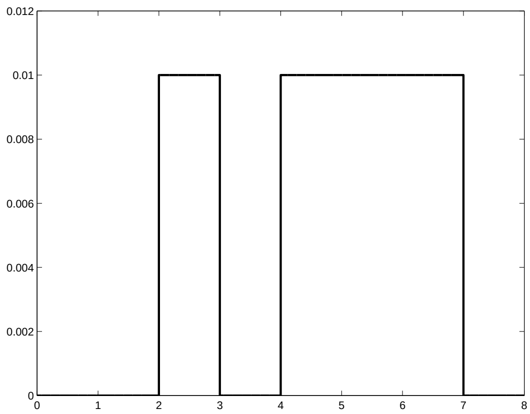

Fig. 13. amplitude $A(t)$, $t \in [0, 8\, sec]$

6 Conclusion

The comparison of the LES and DNS of the neutral backward facing step flow without a manipulation ($A = 0$) to the controlled cases with a fixed frequency of $f = 50\, Hz$ and a Reynolds number of 3000 for an amplitude of $A = 0.01\, u_{inflow}$ gives a decrease of the reattachment length X_r of more than 30%. This agrees with the experimental experiences of Huppertz (1994).
The experimental found decrease of the reattachment length X_r and the development of lateral structures behind the step in the case of a manipulation by a moved boundary were confirmed by the numerical simulations.

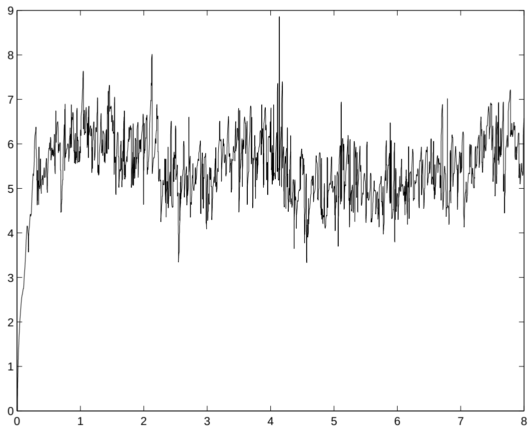

Fig. 14. reattachment length $XR(t)\, t \in [0, 8\, sec]$

Because of the good agreement of the large eddy and direct numerical simulation results with the experimetal data of [1] for qualitative parameter studies the LES will be sufficient.
The results of the unsteady numerical simulations using amplitude functions like in figure 13 with variations of the Reynolds number and A_{max} are suitable for the identification and the construction of robust linear controllers. The results are just summarizing in [9].

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