A Macroscopic Model for Intersecting Pedestrian Streams with Tactical and Strategical Redirection

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Abstract. Intersecting pedestrian flows with distinct intended directions are modeled by a macroscopic model based on a multiphase approach with a convection-diffusion equation. The intersection and the separation of the streams after an intersection is modeled by strategical and tactical components avoiding jams. In particular, we introduce a redirection strategy using dynamic potentials for guiding the pedestrians to their originally intended destination.

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INTRODUCTION

The simulation of pedestrian traffic in situations with many participants like on airports, in railway stations, sport stadiums, public events and manifestations, shopping malls etc. with respect to trouble-free operation and security aspects has been intensively studied in the past years under various aspects. While evacuation scenarios have been understood quite well (see [3], e.g.), the simulation of distinct streams of pedestrians with different destinations still needs attention, in particular, with respect to intersection scenarios. Simulation models are usually classified as microscopic or macroscopic approaches [1]. While microscopic models allow for the simulation of individual behavior and local effects considering pedestrians as individual objects interacting with each other and are based on ODEs, cellular automata or graph theory, macroscopic approaches focus on global, mass effects interpreting pedestrians as particles of flows and use PDE models (cf. [2] or [4] for an overview). Both model types have their specific justification. The main diffulty of macroscopic models results from the fact, that in contrast to standard physical flow situations, pedestrian movement can be directed to any direction and it is strongly influenced by human behaviour. In [5] we considered two variants of a macroscopic multiphase approach based on coupled convection-diffusion equations. The preferred nonlinear model has proved to yield a reliable prediction of congestion situations. In the case of jams, an evasion strategy avoiding (over)crowded regions has been introduced permitting the (re)separation of the originally separated streams and avoiding undirected diffusion using the whole available space. This model still lacks a strategy redirecting the streams towards their originally intended direction after evasion and other unpredictable movements. In the present paper, we complete the model in this sense by guiding pedestrians using the solutions of potential equations reflecting both global and dynamically determined local effects based on geometrical and density information. The model is illustrated by some numerical results.

MODEL

We consider n distinct streams of pedestrians, each stream having an individual intended direction, on an open sufficiently smooth bounded domain Ω in $\mathbb{R}^2$ and an open time interval $(0,T)$. Pedestrian movement can be considered as a transport problem which is principally governed by a mass balance. For $(x,y) \in \Omega$, $t \in (0,T)$, we consider the transport equation

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\vec{v}_i \rho_i) = 0, \quad i = 1,\dots,n, \tag{1}$$

where $\rho_i \equiv \rho_i(x,y,t) \in [0,1]$ and $\vec{v}_i \equiv \vec{v}_i(\rho_1,\dots,\rho_n;x,y,t)$ denote the local pedestrian density and velocity, resp., at time t and position (x,y). This approach does not allow for evasion movements in the case of conflicts of intersecting streams. In [5], we have elaborated this basic model specifying the velocity components v_i by the introduction of a tactical and a strategical component

$$\vec{v}_i = a_i V(\rho)\vec{d}_i^s + b_i W(\rho)\vec{d}^t, \quad i = 1,\dots,n, \tag{2}$$

where $V(\rho)$ denotes an appropriate normalized density-dependent velocity function depending on the total pedestrian density $\rho = \sum_{i=1}^n \rho_i \in [0,1]$ and where $W(\rho) = 1 - V(\rho)$. V is chosen as a monotonically decreasing function in [0,1] with $V(0) = 1, V(1) = 0$. Examples for V are $V(\rho) = 1-\rho$ or $V(\rho) = 1 - e^{h(\rho)}$ with a suitable function h. The choice of a function V can be assisted if application specific knowledge of or assumptions on the principal shape of the flow function $\rho V(\rho)$ are available. Application specific information on the so called, usually one dimensional *fundamental diagram* (see [6], e.g.) can be helpful. The constants a_i correspond to actual maximum pedestrian velocities (typically around 1.4 m/s), the b_i are used as similar weighting constants for the second component. Species specific unit vector fields $\vec{d}_i^s \equiv \vec{d}_i^s(x,y)$ indicate the intended walking directions, $\vec{d}^t \equiv \vec{d}^t(x,y)$ indicates the direction of local correction depending on the total density, thus being common to all streams. The strategic component $\vec{v}_i^s = a_i V \vec{d}_i^s$ contained in the (convective) term $\nabla \cdot \vec{f}_i(\dots)$ reflects the disposition to reach the desired destination on a desired path, while tactical one $\vec{v}_i^t = b_i W \vec{d}_i^t$ allows for avoiding densely populated areas. By setting $V + W = 1$ we assume that both goals are pursued with the same intensity.

The equations (1) and (2) result in a generalized, possibly nonlinear model

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \vec{f}_i(\rho_1,\dots,\rho_n;t,x,y) = \sum_{j=1}^n \nabla \cdot (b_{ij}(\rho_1,\dots,\rho_n)\nabla\rho_j), \quad i = 1,\dots,n, \tag{3}$$

where $\vec{f}_i(\rho_1,\dots,\rho_n;t,x,y) = \vec{v}_i(\rho_1,\dots,\rho_n;t,x,y)\,\rho_i$ and where the $b_{ij} \equiv b_{ij}(\rho_1,\dots,\rho_n)$, $1 \le i,j \le n$, denote the components of a diffusion matrix B. The model needs to be completed by appropriate initial conditions $\rho_i(t=0) = \rho_i^{(0)}$ and suitable boundary conditions like no flux through walls or assumptions on flow or density at entries and exits.

The next, decisive modeling step concerns the choice of suitable direction fields $\vec{d}_i^s, i = 1,\dots,n$ and $\vec{d}^t$. The tactical directions

$$\vec{d}_i^t = \begin{cases} -\nabla\rho/|\nabla\rho| & \text{for} \quad |\nabla\rho| > 1, \\ -\nabla\rho & \text{for} \quad |\nabla\rho| \le 1, \end{cases} \tag{4}$$

introduced in [5] model an evasion movement away from densely populated areas. Defining the function

$$\chi(\rho) = \begin{cases} 1/|\nabla\rho| & \text{if} \quad |\nabla\rho| > 1, \\ 1 & \text{if} \quad |\nabla\rho| \le 1, \end{cases} \tag{5}$$

we obtain (3) with $b_{ij}(\rho_1,\dots,\rho_n) = b_i\,\rho_i\,W\,\chi(\rho)$, $i,j = 1,\dots,n$.

In [5], the strategical directions $\vec{d}_i^s, i = 1,\dots,n$ were modeled as straight linear lines connecting directly to the intended target. Therefore, the combination of strategical and tactical directions only allowed for a combination of a movement on the currently shortest path to the target and an evasion away from crowded regions. The model thus still lacks a realistic redirection towards the intended direction taking into account other important information like geometrical aspects (obstacles inside the area, e.g.) or additional influence factors (current total local density on the intended way possibly causing deviations, e.g.). A significant improvement of the previous model can be obtained by updating the strategical directions $\vec{d}_i^s, i = 1,\dots,n$ from the solutions of appropriate potential equations

$$\begin{aligned} \Delta\phi_j^{(i)}(t) &= r_j^{(i)}(t),\ j = 1,\dots,n \\ \phi_i(t) &= \sum_{j=1}^n \phi_j^{(i)}(t) \\ \vec{d}_i^s &= \begin{cases} \frac{\nabla\phi_i(t)}{|\nabla\phi_i(t)|} & \text{if} \quad |\nabla\phi_i(t)| \neq 0, \\ \text{random unit vector} & \text{if} \quad |\nabla\phi_i(t)| = 0, \end{cases} \end{aligned} \tag{6}$$

for $i = 1, \ldots, n$. The additional information, global or local, geometrical or density dependent, etc. can be integrated into the righthand sides $r_j^{(i)}(t)$. The respective pedestrian stream is now guided using dynamically updated potential lines which can reflect any necessary influence factor. The strategy 6 yields a rather general framework which can be used in many ways.

NUMERICAL EXAMPLES

Numerical tests have been carried out both for standard situations like 90° and 180° intersections of two streams on a square with specified entries and exits in the walls and for a real-life scenario outlined in the sequel. As domain we choose a subregion of roughly $20\,m$ by $10\,m$ of the entry area of the mathematics building at the Technical University of Berlin. The above method has been implemented as a finite volume method with a triangulation depicted in Fig. 1 with triangles having an edge length of approximately $50\,cm$ taking into account the typical space requirement of a pedestrian. As velocity function we chose $V(\rho) = 1 - e^{-\frac{1.913}{5.4(1/\rho - 1)}}$.

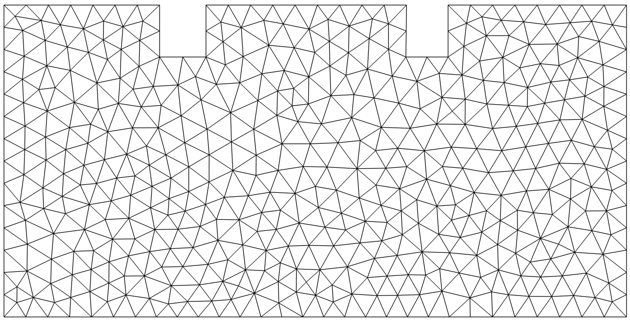

FIGURE 1. Simulation grid

The update procedure 6 for the strategical directions has been implemented

For model verification, we have carried out several life experiments with up to 200 volunteers which were recorded by video capture and evaluated semi-automatically. Fig. 2 shows a snapshot from a 90°- intersection of two groups with head marks. For the macroscopic model we integrated density information derived from the real-life data as boundary information at the two entries.

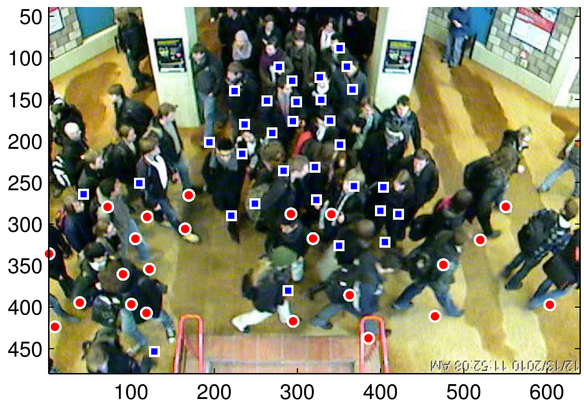

FIGURE 2. Human crowd experiment: pedestrian positions at a specific point in time of group A (red) and group B (blue).

The update procedure 6 for the the strategical directions on which the present paper is focussed, has been implemented with respect to two aspects in this context. First, we intend to redirect pedestrian streams to their strategic destination after possible evasion movements from crowded populated areas by taking into account global information concerning the current total density: Even back on the "right way" we try to circumvent dynamically possibly higher concentrations by defining

$$r_j^{(i)}(t,x,y) = \begin{cases} \rho_i(t,x,y) & \text{if } \rho_i(t,x,y) \geq \tau, \\ 0 & \text{else} \end{cases} \quad i = 1,\dots,n, \tag{7}$$

where τ denotes a threshold indicating a critical density requiring a deviation in order to avoid a slowdown. A typical critical value is 2.7 pedestrians per m^2, e.g.

A second aspects concerns the phase separation, i.e. the (re)separation of the distinct pedestrians streams. We recall our assumption that the streams do not merge after an intersection: the members of every stream try to continue towards their originally intended direction (exit). This can be implemented by parameters expressing an affinity between members of the same stream and a repulsion of members of other streams:

$$r_j^{(i)}(t,x,y) = \sum_{k=1}^{n} \omega_k^{(i)} \rho_i(t,x,y),\ i,j = 1,\dots,n, \tag{8}$$

with suitable parameters

$$\omega_k^{(i)} = \begin{cases} c_{i,k} > 0 & \text{if } i = k \\ c_{i,k} < 0 & \text{if } i \neq k. \end{cases} \tag{9}$$

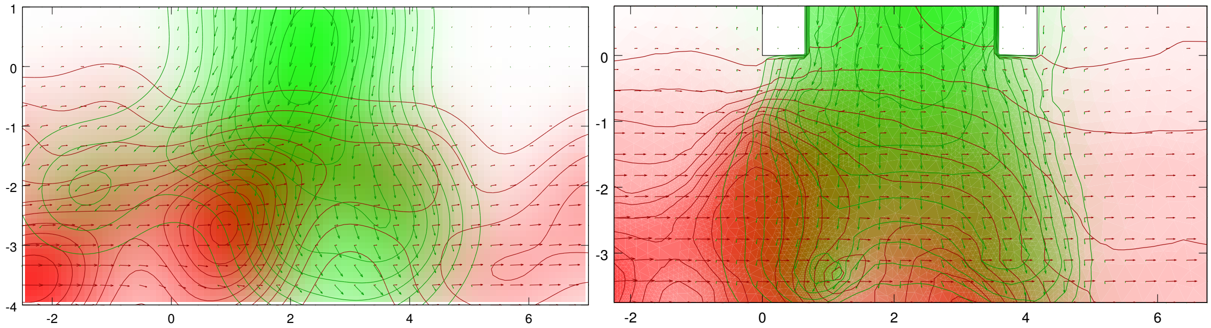

FIGURE 3. Pedestrian densities at time step $t = 70$, red: group A, green: group B. Left: approximated real-world density. Right: macroscopic model (including arrows for the flux).

The left part of Fig. 2 shows a density model at a specific time step derived from the real-life data by applying density estimators based on specifically adapted filters while the right part illustrates the result of the numerical simulation. The necessity of (continuous) density estimators reflects one major problem of macroscopic models. Macroscopic models are per definition continuous models whose validity for inevitably discrete real-life situations is principally coupled with high densities. The latter, however, are not given in many practical applications. In view of the specific properties of macroscopic models it is, however, inevitable to male use of them due to the lack of alternatives, i.e. they must and they can be applied, but with care.

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