The fields K are real quadratic number field with $K = \mathbb{Q}(\sqrt{k})$. If (S_1, S_2) is a Humbert form, where S_1, S_2 are 2×2-real matrices which are positive definite, then we write for

$$S_i = \begin{pmatrix} 1 & b_i \\ b_i & c_i \end{pmatrix} \qquad (i = 1, 2)$$

only (b_1, c_1, b_2, c_2) for short. Eutactic coefficients are denoted with ρ_i where i ranges over the number of minimal vectors of S. The fields are sorted by their discriminants. With e_1 and e_2 we denote the unit vectors $(1,0)^t$ resp. $(0,1)^t$ and with d_P we denote the dimension of the perfection. Altogether this means

$$(S_1^{-1}, S_2^{-1}) = \sum_{i=1}^{n} \rho_i \left(\frac{m_i {m_i}^t}{S_1[m_i]}, \frac{m_i' {m_i'}^t}{S_2[m_i']} \right)$$

where m_i denotes the minimal vectors in the same order as they appear in the table below .

k	abbreviation	entries of S	minimal vectors	eutactic coefficients	d_P	$\frac{m(S)}{\sqrt{\det S}}$
5	$\tau := \frac{1+\sqrt{5}}{2}$	$\left(-\frac{\tau}{2}, 1, -\frac{\tau'}{2}, 1\right)$	$e_1, e_2, (1,1)^t, (\tau,1)^t, (-\tau',1)^t$	$\rho_i = \frac{2}{5}$ $(i = 1,2,3,4,5)$	5	$\frac{4}{\sqrt{5}} = 1.7888\ldots$
2	$\tau := \frac{1+\sqrt{2}}{2}$	$\left(\tau, \frac{\sqrt{6}+\sqrt{2}}{2}, \tau', \frac{\sqrt{6}-\sqrt{2}}{2}\right)$	$e_1, e_2, (-2\tau,1)^t, (-\sqrt{2},1)^t, (-1,1)^t$	$\rho_1 = \frac{1}{15}(14-4\sqrt{6})$, $\rho_i = \frac{1}{15}(4+\sqrt{6})$ $(i = 2,3,4,5)$	5	$\frac{4}{2\sqrt{6}-3} = 2.1063\ldots$
3	$u := 2+\sqrt{3}$	$\left(\frac{u}{2}, u, \frac{u'}{2}, u'\right)$	$e_1, e_2, \left(-u,\ \sqrt{3}\right)^t, \left(1+\sqrt{3},\ -\sqrt{3}\right)^t,$ $\left(-3-2\sqrt{3},\ 1+\sqrt{3}\right)^t, \left(-u,\ 1+\sqrt{3}\right)^t,$ $(-u,1)^t, (-2,1)^t, \left(1+\sqrt{3},\ -1\right)^t,$ $(-1,1)^t, \left(\sqrt{3},\ -1\right)^t, (-u,2)^t$	$\rho_i = \frac{1}{6}$ $(i = 1,\ldots,12)$	5	4.0
13	$\tau_1 := \frac{-8+\sqrt{13}+7\sqrt{7}-2\sqrt{91}}{18}$ $\mu_1 := \frac{7-2\sqrt{13}-14\sqrt{7}+4\sqrt{91}}{9}$ $\tau_2 := \frac{-8-\sqrt{13}-7\sqrt{7}-2\sqrt{91}}{18}$ $\mu_2 := \frac{7+2\sqrt{13}+14\sqrt{7}+4\sqrt{91}}{9}$	$(\tau_1, \mu_1, \tau_2, \mu_2)$	$\left(\frac{-3+\sqrt{13}}{2},\ \frac{-1+\sqrt{13}}{2}\right)^t, \left(\frac{1+\sqrt{13}}{2},\ \frac{3+\sqrt{13}}{2}\right)^t,$ $e_2, \left(1,\ -\frac{3+\sqrt{13}}{2}\right)^t, \left(1,\ \frac{3+\sqrt{13}}{2}\right)^t, e_1$	$\rho_1 = \rho_2 = \frac{9+\sqrt{91}}{28}$, $\rho_3 = \rho_6 = \frac{3(11-\sqrt{91})}{70}$ $\rho_4 = \rho_5 = \frac{29+\sqrt{91}}{140}$	5	$\sqrt{\frac{1476+144\sqrt{91}}{175}} =$ $= 4.03532\ldots$
13	$\tau := \frac{\sqrt{13}-5}{4},\ \mu := \frac{19-5\sqrt{13}}{6}$	(τ, μ, τ', μ')	$e_1, e_2, \left(\frac{-5+\sqrt{13}}{2},\ -2\right)^t, \left(\frac{-3+\sqrt{13}}{2},\ \frac{-1+\sqrt{13}}{2}\right)^t,$ $\left(\frac{1+\sqrt{13}}{2},\ \frac{3+\sqrt{13}}{2}\right)^t, \left(2,\ \frac{5+\sqrt{13}}{2}\right)^t$		3	4.0

k	abbreviation	entries of S	minimal vectors	eutactic coefficients	d_P	$\frac{m(S)}{\sqrt{\det S}}$
17	$\tau_1 := -4+\sqrt{17}-4\sqrt{2}+\sqrt{34}$ $\tau_2 := -4-\sqrt{17}+4\sqrt{2}+\sqrt{34}$	$\left(\frac{1}{2}, \tau_1, \frac{1}{2}, \tau_2\right)$	$(\frac{-5+\sqrt{17}}{2},\ 1)^t, (\frac{-3+\sqrt{17}}{2},\ -1)^t,$ $e_2, (1,-1)^t, e_1$	$\rho_i = \frac{8+\sqrt{34}}{30}$ $i=1,2,3,4,$ $\rho_5 = \frac{14-2\sqrt{34}}{15}$	5	$\sqrt{\frac{784+128\sqrt{34}}{225}} =$ $2.6079\ldots$
17	$\tau_1 := \frac{3+5\sqrt{17}-3\sqrt{5}-\sqrt{85}}{16}$ $\tau_2 := \frac{3-5\sqrt{17}+3\sqrt{5}-\sqrt{85}}{16}$	$\left(\tau_1, \frac{3-\sqrt{5}}{2}, \tau_2, \frac{3+\sqrt{5}}{2}\right)$	$(\frac{3-\sqrt{17}}{2},\ 1)^t, (\frac{5-\sqrt{17}}{2},\ \frac{3-\sqrt{17}}{2})^t, e_2,$ $(1,\ \frac{-3-\sqrt{17}}{2})^t, e_1$	$\rho_1=\rho_4=\frac{12}{45},\ \rho_3=\rho_5=\frac{11+\sqrt{85}}{45}$ $\rho_2=\frac{44-2\sqrt{85}}{45}$	5	$\sqrt{\frac{1408+128\sqrt{85}}{405}} =$ $= 2.5279\ldots$
21	$\tau = \frac{\sqrt{21}-2}{4},\ \mu = \frac{7-\sqrt{21}}{4}$	(τ, μ, τ', μ')	$\left(-1,\ \frac{\sqrt{21}+3}{2}\right)^t, \left(\frac{\sqrt{21}-3}{2},\ -1\right)^t,$ $\left(\frac{\sqrt{21}+1}{2},\ -1\right)^t, \left(\frac{\sqrt{21}-3}{2},\ -2\right)^t,$ $\left(\frac{\sqrt{21}-1}{2}, -2\right)^t, e_1$	$\rho_i = \frac{1}{3}\ (i=1,\ldots,6)$	5	$\frac{16}{3} = 5.3333\ldots$
21	$\tau = \frac{\sqrt{21}+1}{5}, \mu = \frac{\sqrt{21}+11}{10}$	(τ, μ, τ', μ')	$\left(2,\ \frac{-\sqrt{21}+1}{2}\right)^t, \left(-\frac{\sqrt{21}+5}{2},\ \frac{\sqrt{21}+3}{2}\right)^t$ $(-1,\ -1)^t, e_1, (1,\ -1)^t,$ $\left(\frac{\sqrt{21}+3}{2},\ -1\right), \left(\frac{\sqrt{21}+1}{2},\ -2\right)^t, e_2$	$\rho_i = \frac{1}{4}\ (i=1,\ldots,8)$	5	5.0

k	abbreviation	entries of S	minimal vectors	eutactic coefficients	d_P	$\frac{m(S)}{\sqrt{\det S}}$
6	$\tau := \frac{2-2\sqrt{6}}{5},\ \mu := \frac{7-2\sqrt{6}}{5}$	(τ, μ, τ', μ')	$\left(2,\ 1+\sqrt{6}\right)^t,\ \left(1,\ 2+\sqrt{6}\right)^t,$ $\left(-3+\sqrt{6},\ 2-\sqrt{6}\right)^t,\ e_2,$ $\left(2-\sqrt{6},\ -1\right)^t, \left(2+\sqrt{6},\ 3+\sqrt{6}\right)^t,$ $\left(1-\sqrt{6},\ -2\right)^t,\ e_1$	$\rho_i = \frac{1}{4},\ i = 1, ..., 8$	5	5.0
6	$\tau_1 := \frac{15\sqrt{6}-36-12\sqrt{15}+5\sqrt{90}}{6}$ $\tau_2 := \frac{-15\sqrt{6}-36+12\sqrt{15}+5\sqrt{90}}{6}$	$\left(\frac{1}{2}, \tau_1, \frac{1}{2}, \tau_2\right)$	$\left(2-\sqrt{6},\ 1\right)^t,\ (3-\sqrt{6},\ -1)^t,\ e_2,$ $(1,-1)^t, e_1$	$\rho_i = \frac{12+\sqrt{90}}{45}, (i = 1, 2, 3, 4)$ $\rho_5 = \frac{42-4\sqrt{90}}{45}$	5	$\sqrt{\frac{624+64\sqrt{90}}{135}} =$ $= 3.0198\ldots$
6	$\tau := \frac{1+\sqrt{6}}{2},\ \mu := 3+\sqrt{6}$	(τ, μ, τ', μ')	$(1,1)^t,\ (2+\sqrt{6},\ -1)^t,$ $(1-\sqrt{6},\ 3-\sqrt{6})^t, (2-\sqrt{6},\ 3-\sqrt{6})^t,$ $(1+\sqrt{6},\ 2)^t,\ (1,0)^t$		3	4.0

k	abbreviation	entries of S	minimal vectors	eutactic coefficients	d_P	$\frac{m(S)}{\sqrt{\det S}}$
7	$\tau_1 := \frac{25-5\sqrt{7}-5\sqrt{13}+\sqrt{91}}{36}$ $\tau_2 := \frac{25+5\sqrt{7}+5\sqrt{13}+\sqrt{91}}{36}$ $\mu_1 := \frac{2-\sqrt{7}-2\sqrt{13}+\sqrt{91}}{6}$ $\mu_2 := \frac{2+\sqrt{7}+2\sqrt{13}+\sqrt{91}}{6}$	$(\tau_1, \mu_1, \tau_2, \mu_2)$	$(1,\ 2+\sqrt{7})^t,\ e_2,\ (1,-1)^t,\ (2,-1)^t,$ $(2+\sqrt{7},\ 3+\sqrt{7})^t,\ e_1$	$\rho_1 := \frac{4\sqrt{91}}{273},\ \rho_2 := \frac{98+2\sqrt{91}}{140}$ $\rho_3 := \frac{364-34\sqrt{91}}{1365},\ \rho_4 := \frac{26+2\sqrt{91}}{52}$ $\rho_5 := \frac{2\sqrt{91}}{273},\ \rho_6 := \frac{728-68\sqrt{91}}{1365}$	5	$\sqrt{\frac{1476+144\sqrt{91}}{175}} =$ $= 4.03532\ldots$
7	$\tau := \frac{8-20\sqrt{7}}{72}$ $\mu := \frac{4-\sqrt{7}}{3}$	(τ, μ, τ', μ')	$(3-\sqrt{7},\ -2+\sqrt{7})^t,\ (1,1),\ e_2,$ $(2-\sqrt{7},-1)^t,\ (2+\sqrt{7},\ 3+\sqrt{7})^t,\ e_1$	$\rho_1 = \frac{15}{22},\ \rho_2 = \rho_6 = \frac{9}{22},\ \rho_3 = \frac{5}{22},$ $\rho_4 = \rho_5 = \frac{3}{22}$	5	$\frac{36}{11} = 3.2727\ldots$
7	$\tau := \frac{2+\sqrt{7}}{2},\ \mu := \frac{22+8\sqrt{7}}{6}$	(τ, μ, τ', μ')	$(2,\ 2-\sqrt{7})^t,\ (1, 2-\sqrt{7})^t,\ e_2,$ $(2+\sqrt{7},\ -1)^t,\ (2+\sqrt{7},\ 2)^t,\ e_1$		3	4.0
7	$\tau_1 := \frac{-\sqrt{7}-22+\sqrt{15}(5+2\sqrt{7})}{12},$ $\mu_1 := \frac{-3(2+3\sqrt{7})+\sqrt{15}(5+2\sqrt{7})}{6},$ $\tau_2 := \frac{\sqrt{7}-22-\sqrt{15}(5-2\sqrt{7})}{12},$ $\mu_2 := \frac{3(-2+3\sqrt{7})-\sqrt{15}(5-2\sqrt{7})}{6}$	$(\tau_1, \mu_1, \tau_2, \mu_2)$	$\left(-4-\sqrt{7},\ 2+\sqrt{7}\right)^t,\ (1,-1)^t,$ $\left(-2-\sqrt{7},\ 3+\sqrt{7}\right)^t,\ (\sqrt{7},-2)^t,\ e_1$	$\rho_1 = \rho_2 = \frac{19+\sqrt{105}}{80},$ $\rho_3 = \frac{39-3\sqrt{105}}{40}$ $\rho_4 = \rho_5 = \frac{11+\sqrt{105}}{40}$	5	$\sqrt{\frac{2826+270\sqrt{105}}{320}} =$ $= 4.1805\ldots$
7	$\tau_1 := \frac{34+11\sqrt{10}-5\sqrt{7}-4\sqrt{70}}{18}$ $\mu_1 := \frac{-28-17\sqrt{10}+20\sqrt{7}+7\sqrt{70}}{18}$ $\tau_2 := \frac{34-11\sqrt{10}+5\sqrt{7}-4\sqrt{70}}{18}$ $\mu_2 := \frac{-28+17\sqrt{10}-20\sqrt{7}+7\sqrt{70}}{18}$	$(\tau_1, \mu_1, \tau_2, \mu_2)$	$\left(-3-\sqrt{7},\ 2+\sqrt{7}\right)^t,\ (1,-1)^t,$ $(2-\sqrt{7},\ 3-\sqrt{7})^t,\ (-2-\sqrt{7},\ 3+\sqrt{7})^t$ $(\sqrt{7},-2)^t,\ (1,0)^t$	$\rho_1 = \rho_2 = \frac{26-2\sqrt{70}}{55},$ $\rho_3 = \rho_4 = \rho_5 = \rho_6 = \frac{29+2\sqrt{70}}{110}$	5	$\sqrt{\frac{5584+640\sqrt{70}}{605}} =$ $= 4.2521\ldots$

k	abbreviation	entries of S	minimal vectors	eutactic coefficients	d_P	$\frac{m(S)}{\sqrt{\det S}}$
11	$\tau := \frac{8+10\sqrt{11}}{28},\ \mu := \frac{12+\sqrt{11}}{7}$	(τ,μ,τ',μ')	$\left(4+\sqrt{11},-2-\sqrt{11}\right)^t,\ (5+\sqrt{11},-2-\sqrt{11})^t,$ $(2,2-\sqrt{11})^t,\ (-4-\sqrt{11},3+\sqrt{11})^t,$ $(-4+\sqrt{11},-3+\sqrt{11})^t,\ (-2+\sqrt{11},-1)^t,\ (1,-1)^t,$ $(\sqrt{11},-2)^t,\ (-1,4-\sqrt{11})^t,\ (-6-2\sqrt{11},4+\sqrt{11})^t,$ $(1+\sqrt{11},-3)^t,\ e_1$	$\rho_1=\frac{3}{14},\ \rho_2=\frac{8}{35},$ $\rho_3=\frac{37}{630},\ \rho_4=\frac{109}{1260},$ $\rho_5=\frac{11}{140},\ \rho_6=\frac{19}{210},$ $\rho_7=\frac{3}{14},\ \rho_8=\frac{8}{35},$ $\rho_9=\rho_{10}=\frac{4}{35},$ $\rho_{11}=\rho_{12}=\frac{2}{7}$	5	$\frac{28}{3}=9.3333\ldots$
11	$\tau_1 := \frac{-2\sqrt{385}+21\sqrt{11}-7\sqrt{35}+46}{20},$ $\mu_1 := \frac{-2\sqrt{385}+13\sqrt{11}-7\sqrt{35}+53}{5},$ $\tau_2 := \frac{-2\sqrt{385}-21\sqrt{11}+7\sqrt{35}+46}{20},$ $\mu_2 := \frac{-2\sqrt{385}-13\sqrt{11}+7\sqrt{35}+53}{5}$	$(\tau_1,\mu_1,\tau_2,\mu_2)$	$(-4+\sqrt{11},\ -3+\sqrt{11})^t,\ (2,-1)^t,\ (\sqrt{11},-2)^t,$ $(-6-2\sqrt{11},4+\sqrt{11})^t, e_1$	$\rho_1=\frac{490-22\sqrt{385}}{504},$ $\rho_2=\rho_4=\frac{266+10\sqrt{385}}{1008},$ $\rho_3=\rho_5=\frac{42+2\sqrt{385}}{168}$	5	$\sqrt{\frac{16850+850\sqrt{385}}{1296}}=$ $=5.0863\ldots$
11	$\tau := \frac{6+3\sqrt{11}}{10}, \mu := \frac{60+9\sqrt{11}}{35}$	(τ,μ,τ',μ')	$(2,2-\sqrt{11})^t,\ (1,-1)^t,\ (\sqrt{11},-2)^t,$ $(-1,4-\sqrt{11})^t,\ (-6-2\sqrt{11},4+\sqrt{11})^t,\ e_1$	$\rho_1=\rho_6=\frac{29}{254},$ $\rho_2=\rho_4=\frac{170}{381},$ $\rho_3=\rho_5=\frac{335}{762}$	5	$\sqrt{\frac{70000}{1143}}=$ $=7.8257\ldots$
11	$\tau_1 := \frac{\sqrt{715}}{280}+\frac{9\sqrt{11}}{56}+\frac{2\sqrt{65}}{35}+\frac{4}{7},$ $\mu_1 := \frac{\sqrt{715}}{20}+\frac{\sqrt{11}}{4}+\frac{\sqrt{65}}{20}+\frac{1}{4},$ $\tau_2 := \frac{\sqrt{715}}{280}-\frac{9\sqrt{11}}{56}-\frac{2\sqrt{65}}{35}+\frac{4}{7},$ $\mu_2 := \frac{\sqrt{715}}{20}-\frac{\sqrt{11}}{4}-\frac{\sqrt{65}}{20}+\frac{1}{4}$	$(\tau_1,\mu_1,\tau_2,\mu_2)$	$(2,2-\sqrt{11})^t,\ e_2,\ (1,-1)^t,\ (2,-1)^t,$ $(-6-2\sqrt{11},4+\sqrt{11})^t,\ e_1$	$\rho_1=\rho_5=\frac{580+22\sqrt{715}}{2340},$ $\rho_2=\rho_4=\frac{128+2\sqrt{715}}{468},$ $\rho_3=\rho_6=\frac{280-8\sqrt{715}}{585}$	5	$\sqrt{\frac{34880+1280\sqrt{715}}{1053}}=$ $=8.1011\ldots$