19/11

14) $(K, +, \cdot)$ $\quad P \subseteq K$

$a > 0$

(TRI) $\forall a \in K$: entweder $a \in P$ oder $a = 0$
oder $-a \in P$

(VA) $\forall a, b \in K: a \in P \land b \in P \Rightarrow a + b \in P$

(VM) $\forall a, b \in K: a \in P \land b \in P \Rightarrow a \cdot b \in P$

b.1) $\forall a, b, c \in K: a < b \Rightarrow a + c < b + c$

$a < b \Leftrightarrow b - a \in P$ (Def.)

$$b - a = b \underbrace{+ c - c}_{=0} - a = (b - c) + (c - a)$$

$$= \underbrace{(b + c) - (a + c)} \Leftrightarrow a + c < b + c$$

$b - a \in P \Leftrightarrow (b - c) - (a - c) \in P$

b.2) $\forall a, b, c \in K: a < b \land c > 0 \Rightarrow ac < bc$

$a < b \Leftrightarrow b - a > 0$ (Def.)

$(b - a) > 0 \land c > 0 \overset{(VM)}{\Rightarrow} (b - a)c > 0$

$\Rightarrow bc - ac > 0 \Rightarrow ac < bc$ (Def.)

b.3) $\forall a,b,c \in K \quad a<b \wedge c<0 \Rightarrow ac > bc$

$a<b \Rightarrow b-a>0$

$\boxed{c<0 \Rightarrow -c>0}$ (TEI)

$\underset{\text{(VM)}}{(b-a)(-c)>0} \Leftrightarrow -bc+ac>0$

$\Leftrightarrow$ (Punkt b.1) $-bc+ac+bc > bc+0$

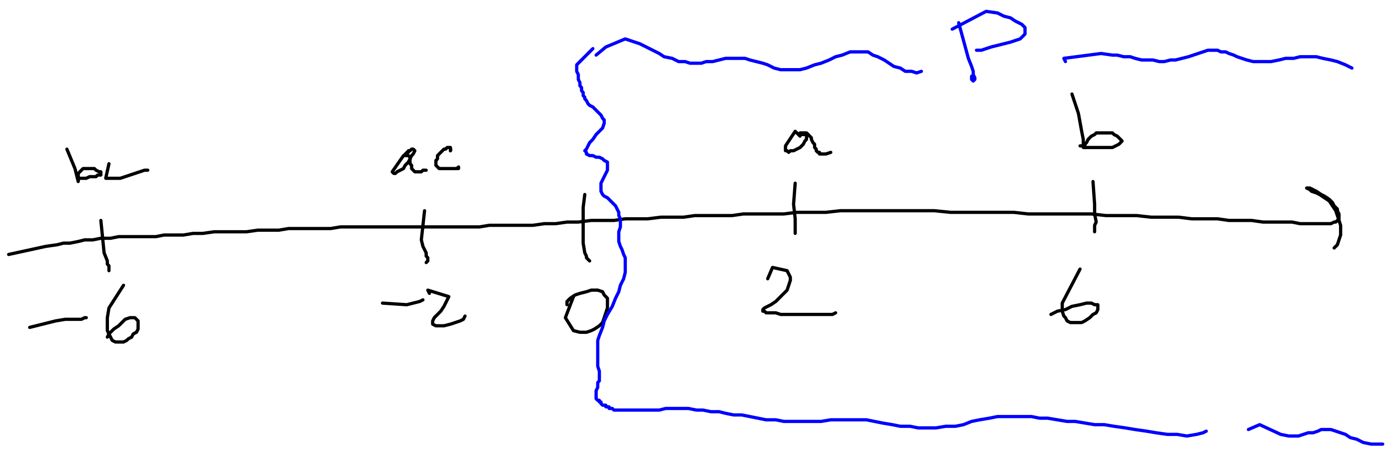
$\Leftrightarrow ac > bc$

$a = 2 > 0$

$b = 6 > 0$

$c = -1 < 0$

$a<b$ $(2<6)$ $\qquad ac>bc$ $(-2>-6)$



iv)

$\boxed{1>0}$

$1 \in K <$

$1<0 \Rightarrow \forall a>0 \Rightarrow a\cdot 1 < 0\cdot 1$

$\Rightarrow a<0$

Widerspruch

$1>0$

$$\boxed{c \in K,\ c > 0 \Rightarrow c^{-1} > 0}$$

c^{-1}:

- $= 0 \Rightarrow c \cdot c^{-1} = 1 = 0$ NEIN!
- $< 0 \Rightarrow c \cdot c^{-1} = 1 < 0$ NEIN!
- > 0

$\forall a, b \in K$

$$0 < a < b \Rightarrow b^{-1} < a^{-1}$$

$0 < 2 < 6$
$\frac{1}{6} < \frac{1}{2}$

$a < b \Rightarrow b - a > 0$

$\Rightarrow b^{-1}(b-a) > 0$ weil $b^{-1} > 0$ und $b - a > 0$

$\Rightarrow b^{-1}(b-a)a^{-1} > 0$

$\Rightarrow \underbrace{b^{-1}b}_{=1}a^{-1} - b^{-1}\underbrace{a a^{-1}}_{=1} > 0 \Rightarrow a^{-1} - b^{-1} > 0$

$\Rightarrow b^{-1} < a^{-1}$

v) $\forall a \in K \setminus \{0\} \Rightarrow a^2 > 0$

a:

- $\cancel{= 0}$
- $> 0 \Rightarrow a \cdot a > 0 \cdot a \Rightarrow a^2 > 0$
- $< 0 \Rightarrow -a \cdot (-a) > 0 \cdot (-a) \Rightarrow a^2 > 0$

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$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$$
$$f(x, y) = (x-1)(y+1)+1$$

$$g : \mathbb{R} \longrightarrow \mathbb{R}^2$$
$$g(u) = (u+1, 2u)$$

$$h_1 = g \circ f$$
$$h_2 = f \circ g$$

a)
$$h_1(3,4) = g(\overbrace{f(3,4)}^{11}) = (12, 22)$$

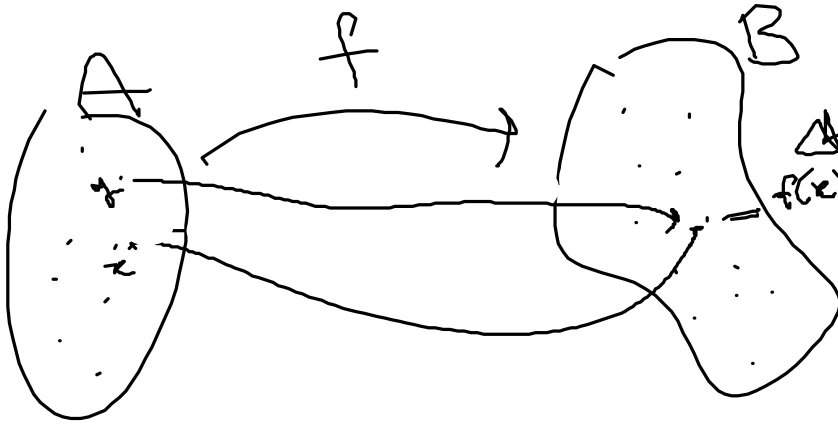
$$h_2(1) = f(g(1)) = f(2,2) = 4$$

$$h_1 \quad \begin{array}{ccccc} \mathbb{R}^2 & \xrightarrow{\quad f \quad} & \mathbb{R} & \xrightarrow{\quad g \quad} & \mathbb{R}^2 \\ (x,y) & \longrightarrow & f(x,y) & \longrightarrow & g(f(x,y)) \end{array}$$

b)
$$h_1(x,y) = g(f(x,y)) = g(\overbrace{(x-1)(y+1)+1}^{u})$$
$$= (\underbrace{(x-1)(y+1)+2}_{u+1}, \underbrace{2[(x-1)(y+1)+1]}_{2u})$$

$$h_2 : \mathbb{R} \xrightarrow{g} \mathbb{R}^2 \xrightarrow{f} \mathbb{R}$$

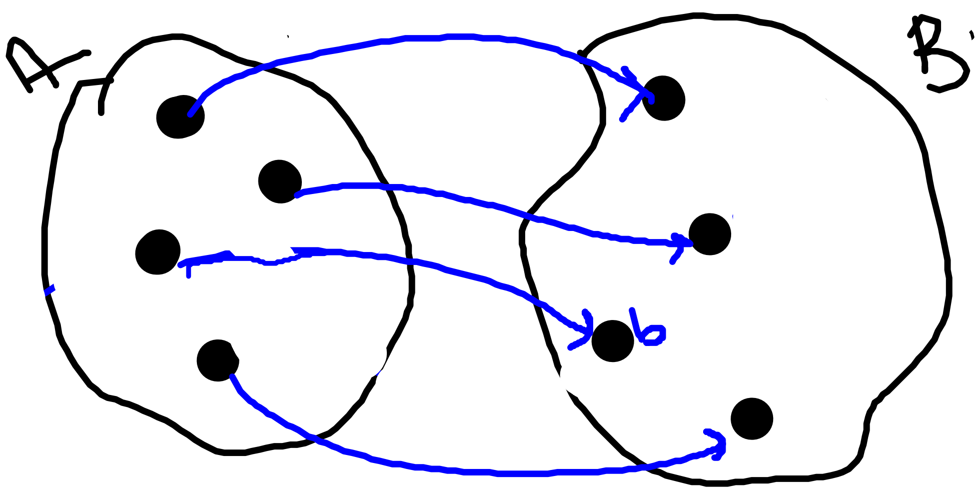
$$h_2(u) = f(g(u)) = f(\overbrace{u+1}^{x}, \overbrace{2u}^{y}) = u(2u+1) + 1 = 2u^2 + u + 1$$



Inj. $\forall x, y \in A : x \neq y \Longrightarrow f(x) \neq f(y)$

$$\Longleftrightarrow$$

$$\forall x, y \in A \quad f(x) = f(y) \Longrightarrow x = y$$



$f^{-1}(\{c\}) = \{\}$

$b \in B \qquad f^{-1}(\{b\}) = \{a \in A : f(a) = b\}$

Urbild von $\{b\}$

Surjektiv:

$$\forall b \in B: \ f^{-1}(\{b\}) \neq \emptyset$$

$$h_2(u) = 2u^2 + u + 1$$

$$h_2^{-1}(\{4\}) = \{u \in \mathbb{R} : h_2(u) = 4\}$$

$$\{u \in \mathbb{R} : 2u^2 + u + 1 = 4\}$$

$$h_2^{-1}(\{4\}) \neq \{\ \}$$

Zum Beispiel $1 \in h_2^{-1}(\{4\})$

$$g(u) = (u+1, 2u)$$

Inj: $g(u_1) = g(u_2) \iff (u_1+1, 2u_1) = (u_2+1, 2u_2)$

$$\iff \begin{matrix} u_1 + 1 = u_2 + 1 \\ 2u_1 = 2u_2 \end{matrix} \iff \boxed{u_1 = u_2}$$

$$f(x,y) = (x-1)(y+1) + 1 \qquad f: \mathbb{R}^2 \longrightarrow \mathbb{R}$$

$u \in \mathbb{R} \qquad f^{-1}(\{u\}) = \{(x,y)\in\mathbb{R}^2 : (x-1)(y+1)+1=u\}$

Surj $\Leftrightarrow$ $f^{-1}(\{u\}) \neq \{\}$

nicht surj $\Leftrightarrow$ $\exists u : f^{-1}(\{u\}) = \{\}$

Ich nehme $u \in \mathbb{R}$ (beliebig)

und setze $y=0$

$$\underbrace{\overset{=x}{(x-1)\cdot 1 + 1}}_{f(x,0)} = u \qquad \boxed{x=u}$$

$$\Rightarrow f(u,0) = u \quad \forall u \in \mathbb{R}$$

$$\Rightarrow (u,0) \in f^{-1}(\{u\}) \Rightarrow f^{-1}(\{u\}) \neq \{\}$$

$$f(x) = x^2 \qquad f: \mathbb{R} \longrightarrow \mathbb{R}$$

$$f^{-1}(-1) = \{x \in \mathbb{R} : x^2 = -1\} = \{\}$$

$$f^{-1}(1) = \{x \in \mathbb{R} : x^2 = 1\} = \{1, -1\}$$
