Theoretical and Numerical Aspects of Controlled Cooling of Steel Profiles

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1 Introduction

The selective intermediate cooling of profiles between the passes of a rolling train results in a temperature equalization between the regions of the cross section with simultaneous reduction of the total heat content. This is a presupposition for the realization of modern technologies, like the normalizing and the thermomechanical rolling, and for the stabilization of the microstructure after the rolling process. Moreover, an accelerated cooling of the profiles from rolling to straightening temperature can replace expensive cooling beds behind the rolling train, so that the investment and working costs are reduced.

Motivated by this background, we considered some basic models for the optimal cooling of steel profiles in cooling sections. A great number of questions concerning modelling, mathematical analysis and numerical methods arose from these investigations. In this paper, we briefly sketch some of them.

2 Mathematical model

In a cooling section, the hot steel profile passes a number of cooling segments, where water is sprayed on its surface. The cooling segments are followed by zones of air cooling, where an equalizing process takes place.

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The scheme of a cooling section is shown in Fig. 1. Let us regard one fixed cross section $\Omega \subset I\!R^2$ of the steel profile during its passage through the section. According to the rolling speed, Ω enters and leaves the particular cooling sections at certain times $0 = t_o \leq t_1 < ... < t_K = T$ $(K = 2k - 1)$ (cf. Fig. 1). Since the heat exchange in axial direction is dominated by that inside Ω, we may adopt the following 2-D model.

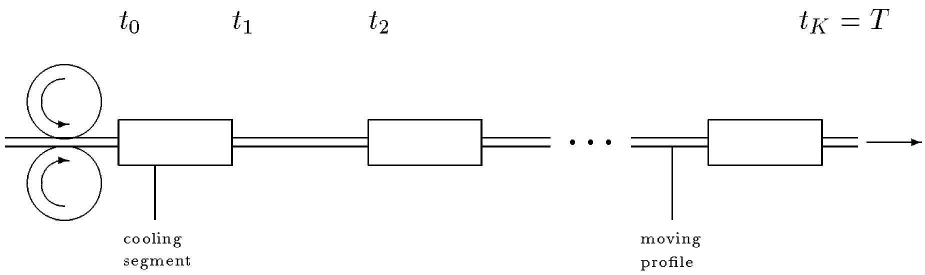

Fig.1: Scheme of a cooling section

In an **air segment** the initial-boundary value problem

$$\begin{aligned} c(\theta(t,x))\rho(\theta(t,x))\,\frac{\partial\theta(t,x)}{\partial t} &= div\,(\lambda(\theta(t,x))\,grad\,\theta(t,x))\,, x \in \Omega \\ \theta(t_{2i-1},x) &= \theta(t_{2i-1}^{-},x) \qquad\qquad\qquad\quad , x \in \Omega \\ \frac{\partial\theta}{\partial n}(t,x) &= 0 \qquad\qquad\qquad\qquad\qquad\quad\; , x \in \Gamma \end{aligned} \tag{2.1}$$

models the heat flow, while in a **cooling segment** the system

$$\begin{aligned} c(\theta(t,x))\rho(\theta(t,x))\,\frac{\partial\theta(t,x)}{\partial t} &= div\,(\lambda(\theta(t,x))\,grad\,\theta(t,x))\,, x \in \Omega \\ \theta(t_{2(i-1)},x) &= \theta(t_{2(i-1)}^{-},x) \qquad\qquad\qquad , x \in \Omega \\ \lambda(\theta(t,x))\,\frac{\partial\theta}{\partial n}(t,x) &= \alpha(x,\theta(t,x))[\theta_{fl} - \theta(t,x)] \quad , x \in \Gamma \end{aligned} \tag{2.2}$$

is regarded $(i = 1, ..., k)$. Inhomogeneous boundary data in (2.1) can be regarded as well. Then the system (2.1) admits the same form and is handled analogously to the system (2.2).

We assume that Ω is a bounded domain with sufficiently smooth boundary Γ. In this setting, $\theta = \theta(t,x)$, $t \in [0,T]$, $x \in \Omega$, is the temperature, $\theta(t^-_{2i-1},x)$, $\theta(t^-_{2(i-1)},x)$ are entrance temperatures obtained from former segments. The functions c, ρ, λ stand for heat capacity , specific gravity, and heat conductivity, respectively. In (2.2), $\alpha = \alpha(x,\theta)$ denotes the heat exchange coefficient, which is strongly dependent on θ, while θ_{fl} denotes the mean temperature of the cooling water.

c, ρ, λ, and α depend on θ and the constituents of the alloy. Some more or less reliable formulas for c, ρ, λ are known, while expressions for α have still to be found. This gives rise to inverse problems, where α is determined numerically from certain measurements. We refer to the contribution by A. Rösch (these Proceedings). The heat capacity c models in particular certain phase changes in the steel.

The system (2.1)-(2.2) belongs to the class of quasilinear parabolic equations. Assuming some smoothness properties of c, ρ, λ, and α, results on existence and uniqueness of θ determined by this system can be found in Ladyzhenskaya and others [3]. We shall not discuss this difficult subject, since the precise behaviour of c is still rather unclear. In the case, where c, ρ, λ are sufficiently smooth positive functions depending only on x, the theory of analytic semigroups applies. Then existence and uniqueness follows from recent results on semilinear equations. We refer to the general exposition in Amann [1] and to Rösch and Tröltzsch [5], where the special semilinear system (2.2) is discussed.

3 Numerical treatment of the heat system

3.1 Rectangular boundaries - splitting up methods

Aiming to control the process optimally, we have to solve the system (2.1)-(2.2) several times, so that a fast solver is needed. For domains consisting of rectangles, like L-shaped domains or double-T beams, we use a local one-dimensional splitting up method. This method is derived from the balance of energy over each time step. For smooth coefficients this scheme is of the order $O(h_x * h_y + \tau)$, where h_x and h_y are the discretisation parameters in direction x and y, respectively, and τ is the time step.

3.2 Curved boundaries - Finite Element Multigrid Method

For domains with curved boundaries the heat equation (2.1)-(2.2) is solved by means of a finite element multigrid method. The discretisation with respect to space was done with help of the programm PREMESH, where the grids are the same for each time step. We use triangular elements and linear test functions. As refinement algorithm the division of each triangle of the corser grid into 4 parts was selected. In the test example (see 4.3) we used 5 grids. Figure 2 shows the second grid. In view of the symmetry of Ω we solved the heat equation only in one half of the domain, prescribing homogeneous Neumann boundary conditions on the symmetry axis. In this reduced domain the finest grid had 22857 nodes and 44800 elements.

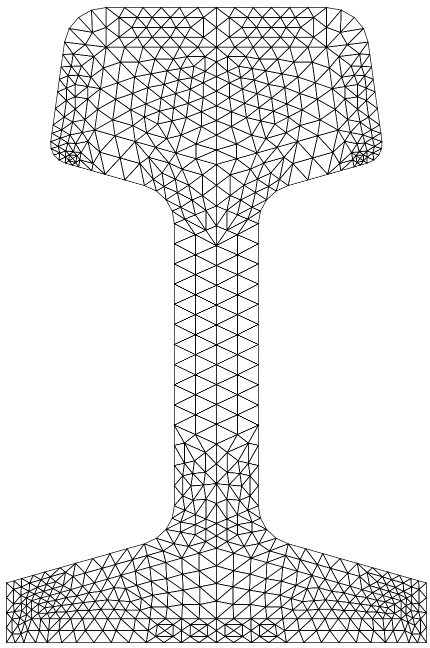

Fig. 2: The second grid

For the time discretisation each interval $(t_{i-1}, t_i), i = 1, ..., K$, is divided into I equal subintervals. For the approximation of the time derivative we used the Crank–Nicholson–scheme with one fully implicite step on the first subinterval, where a jump in the boundary conditions may accure.

4 Optimal control of the cooling process

4.1 Partition of the boundary

We assume that a finite number of spray nozzles is located around the steel profile in each cooling segment. Associated to the location of the nozzles, the boundary Γ is divided into finitely many subdomains $\Gamma_i, i = 1, ..., l$. On Γ_i, $\alpha(x,\theta) = \alpha_i(\theta)$ is assumed to be independent of x (cf. Fig. 3).

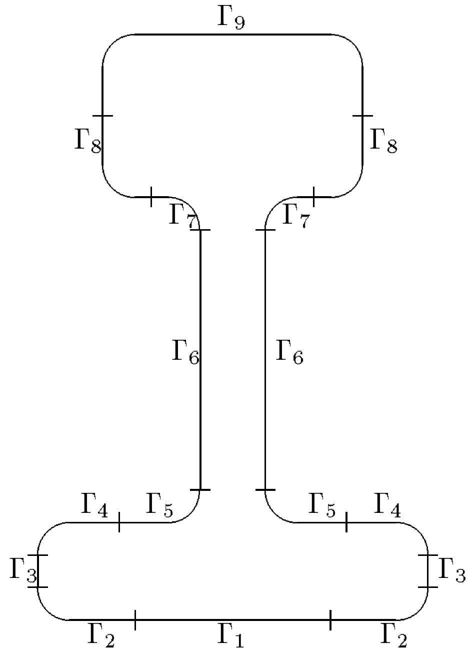

Fig. 3: Partition of Γ in the test example

4.2 The nonlinear optimal control problem

Meaningful optimal control problems can be established in various ways. Very different aspects may be considered for the notion of optimality. This depends on the particular aims of the cooling process. Moreover, the model of the heat system can be simplified for some purposes.

Let us assume that some optimal heat distribution $\bar{\theta}(t,x)$ is prescribed as a reference trajectory, which should be followed as close as possible by the cooling regime. The process is controlled by the water flow through the nozzles directed on the different regions of Γ. On Γ_j we look for the optimal intensity of the flow, expressed through a control function $u_j(t)$. The control may take values in $[0,1]$ (for $u_j = 0$ no water is sprayed on Γ_j, while $u_j = 1$ stands for maximal intensity). The time t corresponds to the position of the cross section Ω in the cooling section (cf. section 2). In this way, we are led to the following optimal control problem:

(P) Minimize the quadratic functional

$$\int_0^T \int_\Omega (\theta(t,x) - \bar{\theta}(t,x))^2 \, dxdt + \nu \int_0^T \int_\Gamma u(t,x)^2 \, dSdt$$

subject to

$$\begin{aligned} c\rho \frac{\partial \theta}{\partial t}(t,x) &= div\,(\lambda \, grad\, \theta(t,x)) & , x \in \Omega \\ \theta(0,x) &= \theta_o(x) & , x \in \Omega \\ \lambda \frac{\partial \theta}{\partial n}(t,x) &= u(t,x)\, \alpha(x,\theta)[\theta_{fl} - \theta(t,x)] & , x \in \Gamma, \end{aligned} \tag{4.3}$$

where $u(\cdot)$ has to be taken from a set $U_{ad} \subset L_\infty((0,T) \times \Gamma)$, which is defined as follows: $U_{ad} = \{u(t,x) \,|\, u(t,x) = 0, t_{2i-1} < t < t_{2i}, u(t,x) = u_j(t), t_{2(i-1)} \le t \le t_{2i-1}, x \in \Gamma_j,$ and $u_j(t) \in [0,1]; i = 1,...,k; j = 1,...,l\}$. The heat exchange function α is defined by $\alpha(x,\theta) = \alpha_j(\theta), x \in \Gamma_j$. We have defined the controls $u_j(t)$ as functions of t. A discretization of u_j with respect to the time arises from the numerical treatment as well as from the limited number of spray nozzles. We shall not discuss this aspect. The parameter $\nu > 0$ regularizes (P) and may be interpreted as the cost for the control ν.

(P) is a fully nonlinear optimal control problem. So far, we did not discuss the problem in this generality. In particular, the concrete form of c and α has still to be determined. The following simplifications may be helpful to get a better understanding of the cooling process. For instance, the boundary condition of (4.3) could be substituted by the simpler one

$$\lambda \frac{\partial \theta}{\partial n}(t,x) = u(t,x)\,[\theta_{fl} - \theta(t,x)]\ , \ x \in \Gamma. \tag{4.4}$$

Then the control system is still nonlinear (bilinear), but independent from the identification of α. Moreover, let us assume that c, ρ, λ depend only on x. Then the optimal control problem belongs to a class of problems governed by semilinear equations, which were discussed in detail during the past years. We refer for instance to Tröltzsch [6] and the references therein.

4.3 A linear control problem

In order to obtain a first idea on the effects of cooling we investigated a very simplified linear version of the problem.

After freezing $c,\ \rho,\ \lambda$ we regarded the problem to

minimize

$$a_1\,\theta(T,z_1) + ...a_M\,\theta(T,z_M)$$

subject to the linear heat equation (4.3) with linear boundary condition

$$-\lambda\,\frac{\partial\theta}{\partial n}(t,x) = q_j(t),\ x\in\Gamma_j,\ t\in[0,T]$$

and to the constraints on *control* q and *state* θ

$$\begin{aligned} 0 \le q_j(t) \le q_{max} &\quad j = 1,...,l \\ q_j(t) = 0, &\quad t\in(t_{2i-1},t_{2i}] \\ \frac{|\theta(t,z_m)-\theta(t,x_n)|}{|z_m-x_n|} \le c_{mn}(t) &\quad m=1,...,M,\ n=1,...,N,\ t\in[0,T]. \\ \frac{|\theta(t,z_1)-\theta(t,z_m)|}{|z_1-z_m|} \le c_m(t) &\quad m=2,...,M,\ t\in[0,T]. \end{aligned}$$

Here, $z_m,\ x_n$ are certain points fixed in Ω, the controls q_j are taken from $L_\infty(0,T)$, and the state constraints are defined by functions $c_{mn}\in C[0,T]$. These constraints are included to approximate a bound for $\nabla\theta$ in order to prevent high stresses in the profile. It should be more realistic to impose bounds on certain components of the stress tensor in Ω.

As an example we regarded a problem with free geometry. That means, no cooling segments and air cooling zones were prescribed (k=1). The following parameters were chosen :

- T = 50 s, [0,T] was divided in 20 subintervals,

- $q_{max} = 10^6$ W/m^2,

- For the state constraints we chose 3 points z_m in the middle of the head, the base and the web of the profile and 9 points x_n on the boundary (M=3, N=9),

- l = 9,

- $c_{mn}(t) = c_m(t) = c(t)$, where $c(t)$ is a step function with $c(0) = 8000^o$C/m, decreasing linearly to $c(25s) = 6000^o$C/m and staying then constant,

- $q_j(t)$ constant on each subinterval of [0,T].

That means, that the functions were chosen as

$$q_j(t) = \sum_{i=1}^{I} q_{ij}\ \varphi_i(t),$$

with some basis functions (step functions) $\varphi_i(t)$. Setting one of the coefficients q_{ij} to 1 and the others to 0, we obtain functions $\theta_{ij}(t,x)$ as solutions of the heat system. Finally there is to solve a linear programming problem for $q_{ij},\ i = 1,...,I,\ j = 1,...,l$. In our test example we had 180 unknowns (I=20, l=9), 180 upper bounds and 580 inequality constraints. Because of the linearity of our problem the solution θ is given as

$$\theta(t,x) = \sum_{i=1}^{I}\sum_{j=1}^{l} q_{ij}\ \theta_{ij}(t,x)$$

For these parameters we found the optimal controls $q_j(t)$, shown in fig. 4.

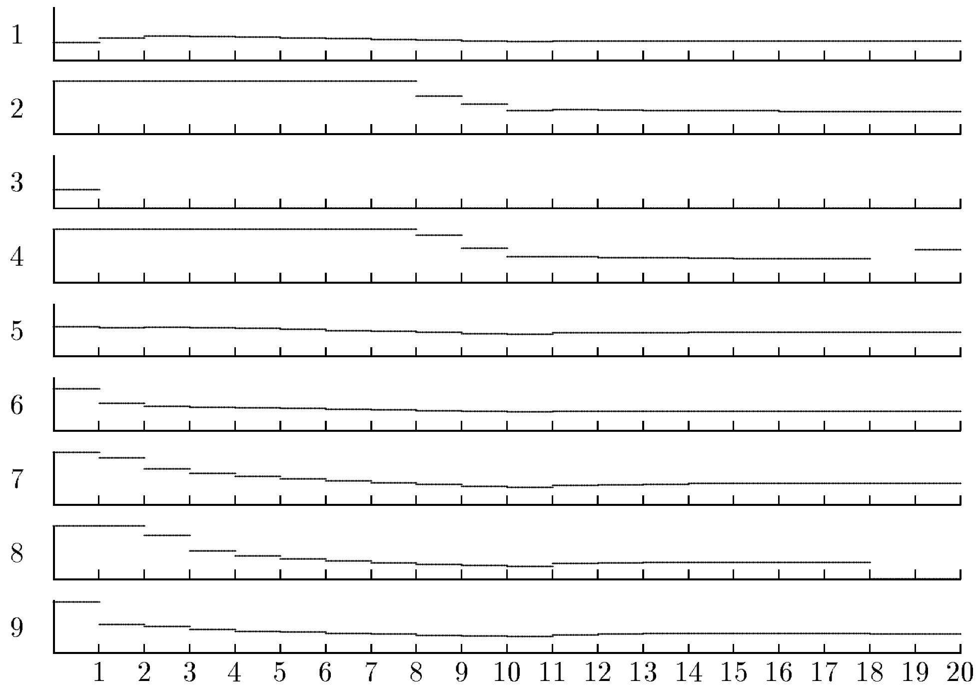

Fig. 4: The optimal controls

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