Mutually diagonal nets on quadrics and incircular nets

Jan Techter TU Berlin September 24, 2019

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	0000	000	0000000	0000	000

A.V. Akopyan, A.I. Bobenko, W.K. Schief, J. Techter On mutually diagonal nets on (confocal) quadrics and 3-dimensional webs, preprint (2019)



A.I. Bobenko, W.K. Schief, Y.B. Suris, J. Techter. On a discretization of confocal quadrics. I. An integrable systems approach, Journal of Integrable Systems (2016) Volume 1:1

A.I. Bobenko, W.K. Schief, Y.B. Suris, J. Techter. On a discretization of confocal quadrics. II. A geometric approach to general parametrization, IMRN (2018)

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
O●O	0000	000	0000000	0000	000

(Checkerboard) incircular nets



- W. Böhm, Verwandte Sätze über Kreisvierseitnetze, Arch. Math. (Basel) 21 (1970) 326–330
- A. Akopyan, A.I. Bobenko, *Incircular nets and confocal conics*, Trans. AMS 370:4 (2018)
- A.I. Bobenko, W.K. Schief, J. Techter. *Checkerboard incircular nets. Laguerre geometry and parametrization*, Geometriae Dedicata (2019)

Previous	work	
000		

Mutually diagonal 000 "Isometric" deformation

Octahedral web

Incircular nets in space form 000

(Checkerboard) incircular nets





W. Böhm, Verwandte Sätze über Kreisvierseitnetze, Arch. Math. (Basel) 21 (1970) 326-330



A. Akopyan, A.I. Bobenko, *Incircular nets and confocal conics*, Trans. AMS 370:4 (2018)

A.I. Bobenko, W.K. Schief, J. Techter. *Checkerboard incircular nets. Laguerre geometry and parametrization*, Geometriae Dedicata (2019)

Previous	work	
000		

Mutually diagonal r 000 "Isometric" deformation 0000000 Octahedral web 0000 Incircular nets in space form 000

(Checkerboard) incircular nets



- W. Böhm, Verwandte Sätze über Kreisvierseitnetze Arch. Math. (Basel) 21 (1970) 326–330

- A. Akopyan, A.I. Bobenko, *Incircular nets and confocal conics*, Trans. AMS 370:4 (2018)
- A.I. Bobenko, W.K. Schief, J. Techter. *Checkerboard incircular nets. Laguerre geometry and parametrization*, Geometriae Dedicata (2019)

Previous work Confocal quadrics Mutually diagonal nets "Isometric" deformations Octahedral webs Incircular nets in s OO● 0000 000 0000 0000 000	space forms
---	-------------





Hilbert

Cohn-Vossen



FIG. 23a

F1G. 23b

	Previous work 00●	Confocal quadrics 0000	Mutually diagonal nets 000	"Isometric" deformations	Octahedral webs 0000	Incircular nets in space forms 000
--	----------------------	---------------------------	-------------------------------	--------------------------	-------------------------	---------------------------------------



Hilbert

Cohn-Vossen

Models at TU Wien

Previous work Contocal quadrics Mutually diagonal nets "Isometric deformations Octahedral webs Incircular nets in space to OC● 0000 0000 0000 0000 0000 0000	Previous work 00●	Confocal quadrics 0000	Mutually diagonal nets 000	"Isometric" deformations	Octahedral webs 0000	Incircular nets in space forr
---	----------------------	---------------------------	-------------------------------	--------------------------	-------------------------	-------------------------------





Hilbert

Cohn-Vossen



00 0000 0000 000000 0000 000	Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space form
	00●	0000	000	0000000	0000	000













Hilbert

Cohn-Vossen

Previous work Contocal guadrics Mutually diagonal nets "Sometric' deformations Octahedral webs Incircular nets in space ⊙OO● OOO0 0000 0000 0000 0000 0000 0000	Previous work	Confocal quadrics 0000	Mutually diagonal nets 000	"Isometric" deformations 0000000	Octahedral webs 0000	Incircular nets in space fo
--	---------------	---------------------------	-------------------------------	-------------------------------------	-------------------------	-----------------------------







Cohn-Vossen



Models at TU Wien

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
00●	0000	000		0000	000





Hilbert

Cohn-Vossen

 Previous work
 Confocal quadrics
 Mutually diagonal nets
 "Isometric" deformations
 Octahedral webs
 Incircular nets in space forms

 Confocal quadrics
 Given a > b > c. Corresponding one-parameter family of confocal quadrics:
 Image: Confocal quadrics
 Image: Confocal quadrics

$$Q(\lambda) = \left\{ (x,y,z) \in \mathbb{R}^3 \mid rac{x^2}{a+\lambda} + rac{y^2}{b+\lambda} + rac{z^2}{c+\lambda} = 1
ight\}, \quad \lambda \in \mathbb{R}.$$



ous work Confocal quadrics

Mutually diagonal 000 "Isometric" deformatio 0000000 Octahedral we 0000 Incircular nets in space forms

Decomposition into three families:

$$\frac{x^2}{u_1 + a} + \frac{y^2}{u_1 + b} + \frac{z^2}{u_1 + c} = 1$$
$$\frac{x^2}{u_2 + a} + \frac{y^2}{u_2 + b} + \frac{z^2}{u_2 + c} = 1$$
$$\frac{x^2}{u_3 + a} + \frac{y^2}{u_3 + b} + \frac{z^2}{u_3 + c} = 1$$

with $-a < u_1 < -b < u_2 < -c < u_3$.

is work Confocal quadrics ○●○○ Mutually diagonal ne 000 "Isometric" deformation

Octahedral we

Incircular nets in space form 000

Decomposition into three families:

$$\frac{x^2}{u_1 + a} + \frac{y^2}{u_1 + b} + \frac{z^2}{u_1 + c} = 1$$
$$\frac{x^2}{u_2 + a} + \frac{y^2}{u_2 + b} + \frac{z^2}{u_2 + c} = 1$$
$$\frac{x^2}{u_3 + a} + \frac{y^2}{u_3 + b} + \frac{z^2}{u_3 + c} = 1$$

with $-a < u_1 < -b < u_2 < -c < u_3$.

This leads to the system of **confocal coordinates** $(x, y, z) = r(u_1, u_2, u_3)$:

$$x^{2} = \frac{(u_{1} + a)(u_{2} + a)(u_{3} + a)}{(a - b)(a - c)}$$
$$y^{2} = \frac{(u_{1} + b)(u_{2} + b)(u_{3} + b)}{(b - a)(b - c)}$$
$$z^{2} = \frac{(u_{1} + c)(u_{2} + c)(u_{3} + c)}{(c - a)(c - b)}$$

Previous work Confocal quadrics Mutually diagonal nets "Isometric" deformations Octahedral webs Incircular nets in space 000 00●0 000 0000000 0000 000	
--	--

For any reparametrization $u_i = u_i(s_i)$ the system of confocal coordinates

has a diagonal first fundamental form:

$$I = d\mathbf{r} \cdot d\mathbf{r} = H_1^2 ds_1^2 + H_2^2 ds_2^2 + H_3^2 ds_3^2,$$

with $\frac{H_i^2}{H_k^2} = \frac{V_i(s_i, s_l)}{V_k(s_k, s_l)},$

Confocal quadrics	Mutually diagonal nets		Incircular nets in space forms
0000			

For any reparametrization $u_i = u_i(s_i)$ the system of confocal coordinates

has a diagonal first fundamental form:

$$\begin{split} \mathbf{I} &= d\mathbf{r} \cdot d\mathbf{r} = H_1^2 ds_1^2 + H_2^2 ds_2^2 + H_3^2 ds_3^2, \\ \text{with } &\frac{H_i^2}{H_k^2} = \frac{V_i(s_i, s_l)}{V_k(s_k, s_l)}, \end{split}$$

has diagonal second fundamental forms:

$$II_{ik} = -d\mathbf{r} \cdot d\mathbf{N}_{ik} = e_{ik}ds_i^2 + g_{ik}ds_k^2,$$

with $\frac{e_{ik}}{g_{ik}} = -\frac{U_i(s_i)}{U_k(s_k)}, \quad U_i = \frac{1}{4}\frac{u_i'^2}{(u_i + a)(u_i + b)(u_i + c)}.$

where $N_{ik} \sim r_{u_l}$ is the unit normal of the quadric $u_l = \text{const.}$

Confocal quadrics	Mutually diagonal nets		Incircular nets in space forms
0000			

For any reparametrization $u_i = u_i(s_i)$ the system of confocal coordinates

has a diagonal first fundamental form:

$$\begin{split} \mathbf{I} &= d\textbf{\textit{r}} \cdot d\textbf{\textit{r}} = H_1^2 ds_1^2 + H_2^2 ds_2^2 + H_3^2 ds_3^2, \\ \text{with } \frac{H_i^2}{H_k^2} &= \frac{V_i(s_i, s_l)}{V_k(s_k, s_l)}, \end{split}$$

has diagonal second fundamental forms:

$$\begin{aligned} \Pi_{ik} &= -d\mathbf{r} \cdot d\mathbf{N}_{ik} = e_{ik} ds_i^2 + g_{ik} ds_k^2, \\ \text{with } \frac{e_{ik}}{g_{ik}} &= -\frac{U_i(s_i)}{U_k(s_k)}, \quad U_i = \frac{1}{4} \frac{u_i'^2}{(u_i + a)(u_i + b)(u_i + c)}. \end{aligned}$$

where $N_{ik} \sim r_{u_l}$ is the unit normal of the quadric $u_l = \text{const.}$

In particular

Orthogonal coordinate system along curvature lines of its isothermal coordinate surfaces.

Previous work	Confocal quadrics	Mutually	/ diagonal nets	"Isometric"	deformations	Octahedral webs	Incircular nets in space forr
000	000●	000		000000	O	0000	000
	$\mathrm{II}_{ik}=e_{ik}ds_i^2+g_i$	$_{ik}ds_k^2,$	$rac{e_{ik}}{g_{ik}} = -$	$-\frac{U_i(s_i)}{U_k(s_k)},$	$U_i=\frac{1}{4}\frac{1}{(u)}$	$\frac{u_i^{\prime 2}}{u_i + a)(u_i + b)(u_i)}$	$\overline{u_i+c)}$.

There exists a unique parametrisation $u_i = u_i(s_i)$ of confocal coordinate lines (up to a scaling of the s_i by the same constant) such that the second fundamental forms of the confocal quadrics are "conformally flat", that is,

$$\text{II}_{12} \sim ds_1^2 + ds_2^2$$
, $\text{II}_{13} \sim ds_1^2 - ds_3^2$, $\text{II}_{23} \sim ds_2^2 + ds_3^2$

There exists a unique parametrisation $u_i = u_i(s_i)$ of confocal coordinate lines (up to a scaling of the s_i by the same constant) such that the second fundamental forms of the confocal quadrics are "conformally flat", that is,

$$\text{II}_{12} \sim ds_1^2 + ds_2^2$$
, $\text{II}_{13} \sim ds_1^2 - ds_3^2$, $\text{II}_{23} \sim ds_2^2 + ds_3^2$

Solve $U_1 = -U_2 = U_3 = \text{const} > 0$

There exists a unique parametrisation $u_i = u_i(s_i)$ of confocal coordinate lines (up to a scaling of the s_i by the same constant) such that the second fundamental forms of the confocal quadrics are "conformally flat", that is,

$$\text{II}_{12} \sim ds_1^2 + ds_2^2$$
, $\text{II}_{13} \sim ds_1^2 - ds_3^2$, $\text{II}_{23} \sim ds_2^2 + ds_3^2$

Solve $U_1 = -U_2 = U_3 = \text{const} > 0 \rightarrow \text{Weierstrass } \wp\text{-function.}$

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	000●	000	0000000	0000	

There exists a unique parametrisation $u_i = u_i(s_i)$ of confocal coordinate lines (up to a scaling of the s_i by the same constant) such that the second fundamental forms of the confocal quadrics are "conformally flat", that is,

$$\mathrm{II}_{12} \sim \textit{ds}_1^2 + \textit{ds}_2^2, \quad \mathrm{II}_{13} \sim \textit{ds}_1^2 - \textit{ds}_3^2, \quad \mathrm{II}_{23} \sim \textit{ds}_2^2 + \textit{ds}_3^2$$

Solve $U_1 = -U_2 = U_3 = \text{const} > 0 \rightarrow \text{Weierstrass } \wp\text{-function.}$

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	000●	000	0000000	0000	

There exists a unique parametrisation $u_i = u_i(s_i)$ of confocal coordinate lines (up to a scaling of the s_i by the same constant) such that the second fundamental forms of the confocal quadrics are "conformally flat", that is,

$$\mathrm{II}_{12} \sim \textit{ds}_1^2 + \textit{ds}_2^2, \quad \mathrm{II}_{13} \sim \textit{ds}_1^2 - \textit{ds}_3^2, \quad \mathrm{II}_{23} \sim \textit{ds}_2^2 + \textit{ds}_3^2$$

Solve $U_1 = -U_2 = U_3 = \text{const} > 0 \quad \rightarrow \quad \text{Weierstrass } \wp\text{-function}.$

In terms of Jacobi elliptic functions this parametrization is given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \sqrt{a-c} \begin{pmatrix} \operatorname{sn}(s_1, k_1) \operatorname{dn}(s_2, k_2) \operatorname{ns}(s_3, k_3) \\ \operatorname{cn}(s_1, k_1) \operatorname{cn}(s_2, k_2) \operatorname{ds}(s_3, k_3) \\ \operatorname{dn}(s_1, k_1) \operatorname{sn}(s_2, k_2) \operatorname{cs}(s_3, k_3) \end{pmatrix},$$

with $k_1^2 = \frac{a-b}{a-c}$, $k_2^2 = \frac{b-c}{a-c} = 1 - k_1^2$, $k_3 = k_1$.

Previous work 000	Confocal quadrics 0000	Mutually diagonal nets ●00	"Isometric" deformations 0000000	Octahedral webs 0000	Incircular nets in space forms 000
Net					
A two-	parameter family	of curves on a surfa	ace, such that there e	exist exactly two	curves of the

family passing through any point on the surface.

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	0000	●00	0000000	0000	000
Net					

A two-parameter family of curves on a surface, such that there exist exactly two curves of the family passing through any point on the surface.



Two diagonally related nets \mathcal{N}_1 ,

For any quadrilateral of \mathcal{N}_1 with one pair of opposite vertices connected by a curve of \mathcal{N}_2 , the other pair of opposite vertices is also connected by a curve of \mathcal{N}_2 .



Two diagonally related nets \mathcal{N}_1 ,

For any quadrilateral of \mathcal{N}_1 with one pair of opposite vertices connected by a curve of \mathcal{N}_2 , the other pair of opposite vertices is also connected by a curve of \mathcal{N}_2 .



Two diagonally related nets $\mathcal{N}_{\mathbf{I}}$,

For any quadrilateral of \mathcal{N}_1 with one pair of opposite vertices connected by a curve of \mathcal{N}_2 , the other pair of opposite vertices is also connected by a curve of \mathcal{N}_2 .

symmetric relation



Two diagonally related nets \mathcal{N}_1 ,

For any quadrilateral of \mathcal{N}_1 with one pair of opposite vertices connected by a curve of \mathcal{N}_2 , the other pair of opposite vertices is also connected by a curve of \mathcal{N}_2 .

- symmetric relation
- If $\mathbf{r}(u, v)$ is a parametrization, then the two nets given by

$$\begin{cases} u = \text{const} \\ v = \text{const} \end{cases} \text{ and } \begin{cases} u + v = \text{const} \\ u - v = \text{const} \end{cases}$$

are diagonally related.

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms 000
200	0000	O●O	0000000	0000	

On any one-sheeted hyperboloid, the lines of curvature and the (straight) asymptotic lines form mutually diagonal nets.



Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
200	0000	O●O	0000000	0000	000

On any one-sheeted hyperboloid, the lines of curvature and the (straight) asymptotic lines form mutually diagonal nets.



Every one-sheeted hyperboloid is part of a confocal family.

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
200	0000	○●○	0000000	0000	000

On any one-sheeted hyperboloid, the lines of curvature and the (straight) asymptotic lines form mutually diagonal nets.



Every one-sheeted hyperboloid is part of a confocal family. Second fundamental form: $II_{13} \sim ds_1^2 - ds_3^2 = (ds_1 + ds_3)(ds_1 - ds_3)$.

 Previous work
 Confocal quadrics
 Mutually diagonal nets
 "Isometric" deformations
 Octahedral webs
 Incircular nets in space forms

 000
 000
 000
 000
 000
 000
 000
 000

What is the meaning of the curvature lines on confocal ellipsoids being parametrized such that

$${\rm II}_{12} \sim ds_1^2 + ds_2^2, \qquad {\rm II}_{23} \sim ds_2^2 + ds_3^2?$$

 Incricular nets
 Mutually diagonal nets
 "Isometric" deformations
 Octahedral webs
 Incricular nets in space for oco

 What is the meaning of the curvature lines on confocal ellipsoids being parametrized such that

 ${\rm II}_{12} \sim ds_1^2 + ds_2^2, \qquad {\rm II}_{23} \sim ds_2^2 + ds_3^2?$







 ${\rm II}_{12} \sim \textit{ds}_1^2 + \textit{ds}_2^2, \qquad {\rm II}_{23} \sim \textit{ds}_2^2 + \textit{ds}_3^2?$







These lines are characteristic conjugate lines:

Previous work Confocal quadrics Mutually diagonal nets of the curvature lines on confocal ellipsoids being parametrized such that

 ${\rm II}_{12} \sim ds_1^2 + ds_2^2, \qquad {\rm II}_{23} \sim ds_2^2 + ds_3^2?$







These lines are characteristic conjugate lines:

analogue of asymptotic lines on positively curved surfaces

Previous work Confocal quadrics Octahedral webs Incircular nets in space form Octahedral webs Octahedral webs

 ${\rm II}_{12} \sim \textit{ds}_1^2 + \textit{ds}_2^2, \qquad {\rm II}_{23} \sim \textit{ds}_2^2 + \textit{ds}_3^2?$







These lines are characteristic conjugate lines:

- analogue of asymptotic lines on positively curved surfaces
- conjugate directions that are bisected by curvature lines

revious work Confocal quadrics Mutually diagonal nets "Isometric" deformations Octahedral webs Incircular nets in space forr

 ${\rm II}_{12} \sim \textit{ds}_1^2 + \textit{ds}_2^2, \qquad {\rm II}_{23} \sim \textit{ds}_2^2 + \textit{ds}_3^2?$







These lines are characteristic conjugate lines:

- analogue of asymptotic lines on positively curved surfaces
- conjugate directions that are bisected by curvature lines

Theorem

On any ellipsoid / two-sheeted hyperboloid, the lines of curvature and the characteristic conjugate lines form mutually diagonal nets.

|--|

Additional property (for asymptotic lines on hyperboloids)

If $P_1, P_3, P_{\overline{1}}, P_{\overline{3}}$ are the vertices of a quadrilateral of asymptotic lines with curvature lines as diagonals, then $\overline{P_1P_3} + \overline{P_{\overline{1}}P_{\overline{3}}} = \overline{P_1P_{\overline{3}}} + \overline{P_{\overline{1}}P_{\overline{3}}}$.



 Previous work
 Confocal quadrics
 Mutually diagonal nets
 "Isometric" deformations
 Octahedral webs
 Incircular nets in space forms

 000
 000
 000
 000
 000
 000

Sphere model of a one-sheeted hyperboloid:



Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	0000	000	○●○○○○○	0000	000

Disk model of a one-sheeted hyperboloid:



Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	0000	000	○●○○○○○	0000	000

Indisk model of a one-sheeted hyperboloid:



Previous work Confo 000 000	cal quadrics Mutually O 000	diagonal nets "Isometric" d OO●OOOO	leformations Octahedral	webs Incircular nets in space forms
--------------------------------	--------------------------------	---	-------------------------	-------------------------------------

The deformation between the one-sheeted hyperboloids of a confocal family

- preserves lines of curvature / asymptotic lines and their mutual diagonal relation
- is isometric along the asymptotic lines
- produces skew parallelograms



Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	0000	000	000●000	0000	000

"Isometric" deformation of hyperboloids

000	

adrics Mu OC

utually diagonal nets

"Isometric" deformations

Octahedral web

Incircular nets in space form 000

Incircular nets



Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	0000	000	00000●0	0000	000

"Isometric" deformation of ellipsoids

Circular sections and "diagonal" curvature lines.

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	0000	000	000000●	0000	000

Hyperbolic incircular net



Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	0000	000		●000	000
Is there a	version of confo	cal coordinates wher	e the curvature lines	on all quadrics a	ire

diagonally related to straight lines?

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	0000	000	0000000	•000	000

Is there a version of confocal coordinates where the curvature lines on all quadrics are diagonally related to straight lines?

 \rightarrow replace (Euclidean) confocal quadrics by **Minkowski confocal quadrics**:

$$\frac{x^2}{\mu+a} + \frac{y^2}{\mu+b} - \frac{z^2}{\mu+c} = 1$$

 Previous work
 Confocal quadrics
 Mutually diagonal nets
 "Isometric" deformations
 Octahedral webs
 Incircular nets in space forms

 DOO
 OOO
 OOOO
 OOO
 OOO
 OOO

Is there a version of confocal coordinates where the curvature lines on all quadrics are diagonally related to straight lines?

 \rightarrow replace (Euclidean) confocal quadrics by **Minkowski confocal quadrics**:

$$\frac{x^2}{\mu+a} + \frac{y^2}{\mu+b} - \frac{z^2}{\mu+c} = 1$$



 Previous work
 Confocal quadrics
 Mutually diagonal nets
 "Isometric" deformations
 Octahedral webs
 Incircular nets in space forms

 000
 000
 0000000
 000
 000
 000

Is there a version of confocal coordinates where the curvature lines on all quadrics are diagonally related to straight lines?

 \rightarrow replace (Euclidean) confocal quadrics by **Minkowski confocal quadrics**:

$$\frac{x^2}{\mu+a} + \frac{y^2}{\mu+b} - \frac{z^2}{\mu+c} = 1$$



All second fundamental forms: ${\rm II}_{ij} \sim ds_i^2 - ds_j^2, ~~i < j$

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space form
000	0000	000		○●○○	000
Octah	edral webs of p	lanes			

The four "diagonal" families of planes of a system of Minkowski confocal coordinates, given by

- $s_1+s_2+s_3=const, \quad s_1+s_2-s_3=const$
- $s_1-s_2+s_3=const, \quad s_1-s_2-s_3=const$

form an octahedral web of planes.



Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space form
000	0000	000	0000000	○●○○	000
Octah	edral webs of p	lanes			

The four "diagonal" families of planes of a system of Minkowski confocal coordinates, given by

 $s_1 + s_2 + s_3 = const$, $s_1 + s_2 - s_3 = const$ $s_1 - s_2 + s_3 = const$, $s_1 - s_2 - s_3 = const$

form an octahedral web of planes.





R. Sauer, Die Raumeinteilungen, welche durch Ebenen erzeugt werden, von denen je vier sich in einem Punkt schneiden, Sitzgsb. der bayer. Akad. der Wiss., math.-naturw. Abt. (1925) 41-56.

W. Blaschke, Topologische Fragen der Differentialgeometrie II. Achtflachgewebe, Math. Z. 28 (1928) 158–160.

Previous work 000	Confocal quadrics 0000	Mutually diagonal nets 000	"Isometric" deformations	Octahedral webs 0000	Incircular nets in space forms

Conical octahedral grids





Previous work 000	Confocal quadrics 0000	Mutually diagonal nets 000	"Isometric" deformations	Octahedral webs	Incircular nets in space forms

Conical octahedral grids





Previous work Confocal quadrics Mutually diagonal nets "Isometric" deformations Octahedral webs Incircular nets in spa 000 0000 0000 0000 000	ace forms
---	-----------

(Checkerboard) incircular nets



Previous		

Mutually diago 000 "Isometric" defo 0000000 Octahedral webs

Incircular nets in space form 000

(Checkerboard) incircular nets



Previous work 000 Mutually dia

"Isometric" deformatio

Octahedral wel

Incircular nets in space forms ●00

(Checkerboard) incircular nets in space forms



A.I. Bobenko, C.O.R. Lutz, H. Pottmann, J. Techter *Laguerre geometry and incircular nets in space forms*, in preparation

Previous work	Confocal quadrics	Mutually diagonal nets	"Isometric" deformations	Octahedral webs	Incircular nets in space forms
000	0000	000		0000	OOO

Hyperbolic checkerboard incircular net

Previous work 000	Confocal quadrics 0000	Mutually diagonal nets 000	"Isometric" deformations	Octahedral webs 0000	Incircular nets in space forms

Thank you!