Technische Universität Berlin
Institut für Mathematik
Sullivan / Knöppel
http://page.math.tu-berlin.de/~sullivan/L/11W/DG2/ WS 11

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 8

(forms, exterior derivative, integration)

due 4.1.2012

Exercise 1 **5 points**

Let $M = \mathbb{R}^3$. Determine which of the following forms are closed ($d\omega = 0$) and which are exact ($\omega = d\theta$ for some θ):

(a) $\omega = yz\,dx + xz\,dy + xy\,dz$,

(b) $\omega = x\,dx + x^2y^2\,dy + yz\,dz$,

(c) $\omega = 2xy^2\,dx \wedge dy + z\,dy \wedge dz$.

If ω is exact, please write down the potential form θ explicitly.

Exercise 2 **5 points**

Let $\omega \in \Lambda^k(M)$ and $X_1, \ldots, X_{k+1} \in \mathfrak{X}(M)$. Show that

$$d\omega\big(X_1, \ldots, X_{k+1}\big) = \sum_{i=1}^{k+1} (-1)^{i-1} X_i\big(\omega\big(X_1, \ldots, \hat{X}_i, \ldots, X_{k+1}\big)\big) + \sum_{i<j} (-1)^{i+j} \omega\big([X_i, X_j], \ldots, \hat{X}_i, \ldots, \hat{X}_j, \ldots, X_{k+1}\big)$$

(where the caret means that the term is omitted).

Exercise 2 **5 points**

Let $b > a > 0$. Denote by T^2 the torus in $\mathbb{R}^3$ obtained by rotating a circle of radius a in the xz–plane with center at $(b, 0, 0)$ around the z–axis. Determine the volume form $d\,\mathrm{vol}$ corresponding to the induced metric with respect to suitable coordinates and use it to determine $\mathrm{vol}\,(T^2)$.