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http://page.math.tu-berlin.de/~sullivan/L/11W/DG2/ WS 11

# Differential Geometry II: Analysis and Geometry on Manifolds

## Exercise Sheet 5

(Lie bracket)

due 23.11.2011

**Exercise 1** **5 points**

Let $X, Y \in \mathfrak{X}(\mathbb{R}^2)$ be given by

$$X = y\frac{\partial}{\partial x}, \qquad Y = \frac{\partial}{\partial y}.$$

Find the flow with infinitesimal generator $X$ and use it to calculate $L_X Y$ from the definition. Compare with $[X, Y]$ calculated directly. Do the same with the roles of $X$ and $Y$ exchanged.

**Exercise 2** **5 points**

Suppose $(U, \phi)$ is a coordinate neighborhood on $M$, $X, Y \in \mathfrak{X}(M)$, and $E_1, \ldots, E_n$ the corresponding coordinate frames. Note that $[E_i, E_j] = 0$ on $U$. If $X = \sum_i \alpha_i E_i$ and $Y = \sum_j \beta_j E_j$ on $U$, then show that

$$[X, Y] = \sum_{i,j} \left( \alpha_i \frac{\partial \beta_j}{\partial x_i} - \beta_i \frac{\partial \alpha_j}{\partial x_i} \right) E_j$$

on $U$.

**Exercise 3** **5 points**

Show that $(L_X Y)_p$ depends on the fact that we use vector fields, i.e. if $X, \tilde{X}$ agree at the point $p$ but are not identical vector fields, then $(L_X Y)_p$ may differ from $(L_{\tilde{X}} Y)_p$.