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http://page.math.tu-berlin.de/~sullivan/L/11W/DG2/ WS 11

# Differential Geometry II: Analysis and Geometry on Manifolds

## Exercise Sheet 12

(geodesics, curvature forms)

due 1.2.2012

**Exercise 1** **5 points**

Let the upper half plane be considered as a manifold covered by a single coordinate system $M = \{(x_1, x_2) \mid x_2 > 0\}$ with $U = M$ and coordinates $(x_1, x_2)$. If $g_{ij}(x) = (x_2)^{-2}\delta_{ij}$, find $\Gamma_{ij}^k$ and show that if suitably parametrized, the curves $x_1 = \textit{constant}$ are geodesics.

**Exercise 2** **5 points**

Show that two isometries $F_1, F_2 \colon M \to M$ on a Riemannian manifold which agree at a point $p$ and induce the same linear mapping from $T_pM$ agree on a neighborhood of $p$.

**Exercise 3** **5 points**

Define a connection on $M := \mathbb{R}^3$ by setting

$$\Gamma_{12}^3 = \Gamma_{23}^1 = \Gamma_{31}^2 = 1, \Gamma_{21}^3 = \Gamma_{32}^1 = \Gamma_{13}^2 = -1,$$

and all other Christoffel symbols to zero. Show that this connection is compatible with the Euclidean metric, but it is not symmetric. Compute the connection 1–forms $\omega_i^j$ and curvature 2–forms $\Omega_i^j$ with respect to the canonical orthonormal basis.