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http://page.math.tu-berlin.de/~sullivan/L/11W/DG2/ WS 11

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 11

(curvature, geodesics)

due 25.1.2012

Exercise 1 **5 points**

For $k \in \mathbb{R}$, let $M_k := \{p \in \mathbb{R}^2 \mid 1 + k|p|^2 > 0\}$. Define a Riemannian metric on M_k by $g_k(p) := 4\left(1 + k|p|^2\right)^{-2} g_0$, where g_0 denotes the canonical metric on $\mathbb{R}^2$. Show that the sectional curvature on $(M_k; g_k)$ is constant and equal to k. Are these manifolds isometric to each other?

Exercise 2 **5 points**

Show that for any orthonormal basis $F_1, \ldots, F_n$ at $p \in M$, the value of $S(X_p, Y_p) := \sum_{i=1}^{n} R(F_{ip}, X_p, Y_p, F_{ip})$ is independent of the choice of the orthonormal basis and that this formula defines a C^∞–tensor field $S(X, Y)$. Verify that $S(X, Y) = S(Y, X)$.

Exercise 3 **5 points**

Let $M := \{(x, y, z) \mid x^2 + y^2 = 1\} \subset \mathbb{R}^3$ with induced metric. Show that the geodesics on M are curves which have constant angle with the z–axis (i.e. helices, circles and straight lines). Find how many geodesics connect two given points p and q.