TECHNISCHE UNIVERSITÄT BERLIN
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http://page.math.tu-berlin.de/~sullivan/L/11W/DG2/ WS 11

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 1

(Manifolds and examples)

due 26.10.2011

Exercise 1 **5 points**

Let X be a topological space, $x \in X$ and $n \geq 0$. Show that the following statements are equivalent:

i) There is a neighborhood of x which is homeomorphic to $\mathbb{R}^n$.

ii) There is a neighborhood of x which is homeomorphic to an open subset of $\mathbb{R}^n$.

Exercise 2 **5 points**

Let $n \geq 0$ and let $J = \{0, \dots, n\}$. We define $\left(U_j^\pm, \varphi_j^\pm\right)_{j \in J}$ by

$$U_j^\pm := \{x = (x_0, \dots, x_n) \in \mathbb{S}^n \mid \pm x_j > 0\}$$

and

$$\varphi_j^\pm \colon U_j^\pm \to \mathbb{R}^n, \qquad (x_0, \dots, x_n) \mapsto (x_0, \dots, x_{j-1}, x_{j+1}, \dots, x_n)\,.$$

Show that $\left(U_j^\pm, \varphi_j^\pm\right)_{j \in J}$ is a C^∞ atlas for $\mathbb{S}^n$.

Exercise 3 **5 points**

For $\theta \in \mathbb{R}$ we define $U_\theta := \mathbb{S}^1 \setminus \{(\cos\theta, \sin\theta)\} \subset \mathbb{S}^1$ and

$$\varphi_\theta \colon U_\theta \to (\theta, \theta + 2\pi) \subset \mathbb{R}, \qquad (\cos\rho, \sin\rho) \mapsto \rho.$$

i) Explain why this is well–defined and show that $(U_\theta, \varphi_\theta)_{\theta\in\mathbb{R}}$ is a C^∞ atlas for $\mathbb{S}^1$.

ii) Show that

$$\{(x_0, x_1) \in \mathbb{S}^1 \mid x_1 > 0\} \to \mathbb{R}, \qquad (x_0, x_1) \mapsto \frac{x_0}{x_1}$$

is another chart for $\mathbb{S}^1$ and decide whether it is in the C^∞ structure defined by the atlas $(U_\theta, \varphi_\theta)_{\theta\in\mathbb{R}}$.