

Exercise Sheet 6

To review before the test on 29 November 2010

Exercise 1:

Suppose U and V are open sets in \mathbb{R}^2 with $U \cup V$ connected and $H_1(U \cup V) = 0$.
Show that $H_1(U \cap V)$ is isomorphic to $H_1(U) \oplus H_1(V)$.

Exercise 2:

Suppose A and B are disjoint closed subset of \mathbb{R}^2 .

Show that $H_1(\mathbb{R}^2 \setminus (A \cup B))$ is isomorphic to $H_1(\mathbb{R}^2 \setminus A) \oplus H_1(\mathbb{R}^2 \setminus B)$.

If $\mathbb{R}^2 \setminus A$ has m components and $\mathbb{R}^2 \setminus B$ has n components, show that $\mathbb{R}^2 \setminus (A \cup B)$ has $m + n - 1$ components.

Exercise 3:

Suppose $U \subset \mathbb{R}^2$ is open, and $K \subset U$ is compact.

Show $H_1(U \setminus K) \cong H_1 U \oplus H_1(\mathbb{R}^2 \setminus K)$.