Exercise Sheet 6
To review before the test on 29 November 2010

Exercise 1:
Suppose $U$ and $V$ are open sets in $\mathbb{R}^2$ with $U \cup V$ connected and $H_1(U \cup V) = 0$. Show that $H_1(U \cap V)$ is isomorphic to $H_1(U) \oplus H_1(V)$.

Exercise 2:
Suppose $A$ and $B$ are disjoint closed subset of $\mathbb{R}^2$. Show that $H_1(\mathbb{R}^2 \setminus (A \cup B))$ is isomorphic to $H_1(\mathbb{R}^2 \setminus A) \oplus H_1(\mathbb{R}^2 \setminus B)$. If $\mathbb{R}^2 \setminus A$ has $m$ components and $\mathbb{R}^2 \setminus B$ has $n$ components, show that $\mathbb{R}^2 \setminus (A \cup B)$ has $m + n - 1$ components.

Exercise 3:
Suppose $U \subset \mathbb{R}^2$ is open, and $K \subset U$ is compact. Show $H_1(U \setminus K) \cong H_1(U) \oplus H_1(\mathbb{R}^2 \setminus K)$. 