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Topology
WS 10/11

Exercise Sheet 5

Due in tutorials on 24 November 2010

Exercise 1 (6 pts):

A set $U \subset \mathbb{R}^2$ is called *star-shaped* from $p \in U$ if for every other point $q \in U$ the segment $\overline{pq}$ lies entirely in U.
Show that if U is star-shaped (from some point) then $H_1(U) = 0$, that is, every 1-cycle is a 1-boundary.

Exercise 2 (7 pts):

Suppose $U = \mathbb{R}^2 \setminus \{p_1, \ldots, p_n\}$. Consider the map:

$$\begin{aligned} C_1(U) &\to \mathbb{Z}^n \\ \gamma &\mapsto (W(\gamma, p_1), \ldots, W(\gamma, p_n)) \end{aligned}$$

Show that it vanishes on boundaries and thus induces a map: $\phi : H_1(U) \to \mathbb{Z}^n$.
Show that ϕ is an isomorphism.

Exercise 3 (7 pts):

Suppose U and V are open subset of $\mathbb{R}^2$ and $F : U \to V$ a continuous map. For a 1-chain $\gamma = \Sigma n_i \gamma_i$ in U we define,

$$F_*\gamma := \Sigma n_i (F \circ \gamma_i)$$

a 1-chain in V. Analogously we define for 0-chains: $F_* \Sigma n_i p_i := \Sigma n_i F(p_i)$.
Show that $F_* \partial \gamma = \partial F_* \gamma$ for all 1-chains γ, and thus F_* maps 1-cycles to 1-cycles.
Lastly, show that F_* maps 1-boundaries to 1-boundaries.