

Exercise Sheet 3

Due in tutorials on 10 November 2010

Exercise 1:

We say that a space X has the *fixed-point property* if every map $f : X \rightarrow X$ has a fixed point. Show that if X has the fixed-point property and $f : X \rightarrow Y$ is a retraction onto a subspace $Y \subset X$, then also Y has the fixed point property.

Exercise 2:

If $f : \mathbb{S}^1 \rightarrow \mathbb{R}^2 \setminus \{0\}$ has $W(f, 0) = 0$, show that f can be extended to a continuous map $\bar{f} : D^2 \rightarrow \mathbb{R}^2 \setminus \{0\}$.

(Hint: Each path with winding number zero, such as f , is homotopic to a constant path.)

Exercise 3:

Suppose $f : \mathbb{S}^1 \rightarrow \mathbb{S}^1$ has no fixed point. Show that $\deg(f) = 1$, and thus that f is surjective.

Exercise 4:

Suppose $f : \mathbb{S}^1 \rightarrow \mathbb{S}^1$ is *even* in the sense that $f(-p) = f(p)$. Show that $\deg(f)$ is even.

Exercise 5:

Suppose $f : \mathbb{S}^1 \rightarrow \mathbb{S}^1$ never has the same value at two antipodal points (that is, $f(-p) \neq f(p)$ for all p). Show that f is surjective.