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Topology
WS 10/11

Exercise Sheet 15

Due in tutorials on 16 February 2011

Exercise 1 (5 pts):

Let K be the following CW-complex: the one-skeleton is the wedge of $2g$ circles and the attaching map for the single two-cell is given by

$$a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g b_g a_g^{-1} b_g^{-1}.$$

Compute its (cellular) homology and its Euler characteristic. Use the classification of surfaces to conclude that K is the closed orientable surface Σ_g (a sphere with g handles).

Exercise 2 (5 pts):

Let K be the following CW-complex: the one-skeleton is the wedge of h circles and the attaching map for the single two-cell is given by $a_1^2 \cdots a_h^2$. Compute its (cellular) homology and its Euler characteristic. Use the classification of surfaces to conclude that K is the closed nonorientable surface N_h (a sphere with h cross-caps).

Exercise 3 (5 pts):

Suppose X is a finite CW-complex and Y is a k-sheeted covering of X. Show that the Euler characteristics satisfy $\chi(Y) = k\chi(X)$.

Exercise 4 (5 pts):

Fact: Any nonorientable surface has a unique orientable double cover. (You need not prove this.) What is the orientable double cover of N_h? Now suppose X is a k-fold cover of Σ_g. Identify the space X.