Exercise 1 (5 pts):
Let $K$ be the following CW-complex: the one-skeleton is the wedge of $2g$ circles and the attaching map for the single two-cell is given by
\[ a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g b_g a_g^{-1} b_g^{-1}. \]
Compute its (cellular) homology and its Euler characteristic. Use the classification of surfaces to conclude that $K$ is the closed orientable surface $\Sigma_g$ (a sphere with $g$ handles).

Exercise 2 (5 pts):
Let $K$ be the following CW-complex: the one-skeleton is the wedge of $h$ circles and the attaching map for the single two-cell is given by $a_1^2 \cdots a_h^2$. Compute its (cellular) homology and its Euler characteristic. Use the classification of surfaces to conclude that $K$ is the closed nonorientable surface $N_h$ (a sphere with $h$ cross-caps).

Exercise 3 (5 pts):
Suppose $X$ is a finite CW-complex and $Y$ is a $k$-sheeted covering of $X$. Show that the Euler characteristics satisfy $\chi(Y) = k \chi(X)$.

Exercise 4 (5 pts):
Fact: Any nonorientable surface has a unique orientable double cover. (You need not prove this.) What is the orientable double cover of $N_h$? Now suppose $X$ is a $k$-fold cover of $\Sigma_g$. Identify the space $X$. 