

Prof. Dr. John M. Sullivan

Silvia De Toffoli

www.math.tu-berlin.de/~sullivan/L/10W/Top/

Topology

WS 10/11

Exercise Sheet 14

Due in tutorials on 9 February 2011

Exercise 1 (5 pts):

Prove the axiom of exactness for the reduced homology, that is, prove that the following sequence is exact:

$$\dots \rightarrow \tilde{H}_p A \rightarrow \tilde{H}_p X \rightarrow \tilde{H}_p(X, A) \rightarrow \tilde{H}_{p-1} A \rightarrow \dots$$

Exercise 2 (5 pts):

Suppose that A is a nonempty subset of X and that A is acyclic. (Remember, this means that the reduced homology of A vanishes.) Show $H_p(X, A) \cong \tilde{H}_p(X)$.

Exercise 3 (5 pts):

Prove the strong form of the Five Lemma. (See Bredon IV.5.10 or our discussion on 2.Feb.) That is, show that if f_2 and f_4 are surjective and f_5 is injective, then f_3 is surjective. Symmetrically, if f_2 and f_4 are injective, and f_1 is surjective, then f_3 is injective.

Exercise 4 (5 pts):

Describe a cell decomposition as CW-complex for the n -dimensional torus, which is defined as the topological product space of n circles: $\mathbb{T}^n := \mathbb{S}^1 \times \dots \times \mathbb{S}^1$.