Exercise Sheet 13
Due in tutorials on 2 February 2011

In these days the angel of topology and the devil of abstract algebra fight for the
soul of each individual mathematical domain.

*Hermann Weyl, 1939*

**Exercise 1 (5 pts):**
If \( p : Y \to X \) is a covering, then the induced map on fundamental groups is injective. How
about the induced map on \( H_1 \)? Prove or give a counter-example.

**Exercise 2 (5 pts):**
Let \( X \) be the discrete two-point space. Calculate the homology groups \( H_*(X) \).

**Exercise 3 (5 pts):**
Check the functoriality of the homology functor \( H_* \).
That is, if \( f : X \to Y \) and \( g : Y \to Z \) are maps of topological spaces, then the induced maps on
homology satisfy \((f \circ g)_* = f_* \circ g_*\).

**Exercise 4 (5 pts):**
Prove from the Eilenberg–Steenrod axioms that for any space \( X \) we have \( H_p(X, X) = 0 \) for
all \( p \).