Exercise 1 (5 pts): Let $P = \mathbb{S}^2 / \pm$ be the projective plane. Let $U = P \setminus \{p\}$ be the complement of a single point in $P$. ($U$ is homeomorphic to a Möbius band.) What is $\pi_1(U)$?

Exercise 2 (15 pts):
Suppose $X$ is the figure-eight space, consisting of circles $A$ and $B$ joined at the basepoint $x_0$. Its fundamental group is $\pi_1(X, x_0) = \langle A \rangle \ast \langle B \rangle \cong \mathbb{Z} \ast \mathbb{Z}$.

1. Consider the infinite cyclic subgroup $H < \pi_1(X)$ generated by the element $A^2$. What is the covering space of $X$ corresponding to this subgroup $H$? (Don’t try to give a rigorous proof. Just sketch the space and explain why your sketch is correct.) What is the automorphism group of this cover?

2. Now consider the map from $\pi_1(X)$ to the abelian group \( \mathbb{Z}/3 \oplus \mathbb{Z}/3 \) which takes $A$ to $(1, 0)$ and $B$ to $(0, 1)$. The kernel of this map is a normal subgroup $K$ of $\pi_1$. What is the cover of $X$ corresponding to this subgroup $K$? (Again, sketch the cover and explain why it corresponds to this kernel, without giving a full proof.) What is the automorphism group of this cover?