TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Prof. Dr. John M. Sullivan
Silvia De Toffoli
www.math.tu-berlin.de/~sullivan/L/10W/Top/

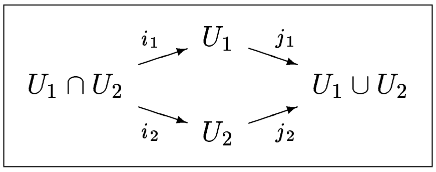
Topology
WS 10/11

Exercise Sheet 11

Due in tutorials on 19 January 2011

Exercise 1 (5 pts):

Consider the categories Top and Top_*.
Show that if U_1 and U_2 are open in $U_1 \cup U_2$ then the following diagram of inclusion maps is a *pushout*.



Exercise 2 (5 pts):

Prove that the following groups are isomorphic:

$$\langle x, y \mid xyx = yxy \rangle \simeq \langle a, b \mid a^2 = b^3 \rangle$$

Exercise 3 (5 pts):

Let G be the subgroup of isometries of the plane generated by the translation: $(x, y) \mapsto (x+1, y)$ and the glide reflection $(x, y) \mapsto (-x, y+1)$.
Show that the action is even. The quotient space $\mathbb{R}^2/G$ is called the *Klein bottle*, K.

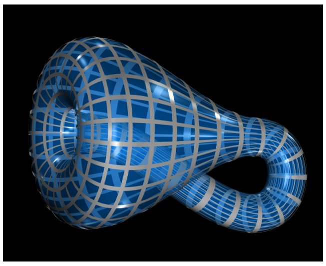


Abbildung 1: Klein bottle

Exercise 4 (5 pts):

Let a group G act evenly on a space Y and let H be a subgroup of G. Show that H also acts evenly on Y and that the natural map $Y/H \to Y/G$ is a covering map. (This is actually a n-sheeted covering, where n is the index of H in G.)

Consider now the group G that we used to define the Klein bottle, and let H be the subgroup the one generated by the following translations of the plane: $(x, y) \mapsto (x+1, y)$, $(x, y) \mapsto (x, y+2)$. What is the space $\mathbb{R}^2/H$? Describe the covering $\mathbb{R}^2/H \to K$.