

Exercise Sheet 10

Due in tutorials on 12 January 2011

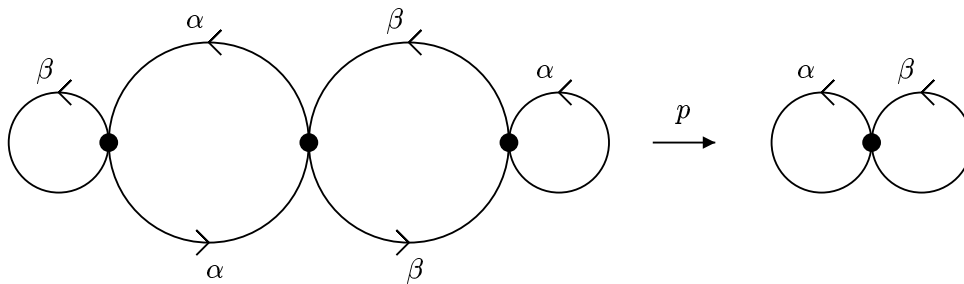
Exercise 1 (5 pts):

Suppose $p : (Y, y_0) \rightarrow (X, x_0)$ is a covering, and suppose α and β are two paths in X from x_0 to some point x_1 . Let $\tilde{\alpha}$ and $\tilde{\beta}$ be their lifts to paths in Y starting at y_0 .

Show that $\tilde{\alpha}$ and $\tilde{\beta}$ have the same endpoint if and only if $[\alpha * \bar{\beta}] \in \pi_1(X, x_0)$ is in $p_*(\pi_1(Y, y_0))$.

Exercise 2 (5 pts):

Recall the covering space from assignment 7:



- What is its group of deck transformations?
- Is this covering regular?

Exercise 3 (5 pts):

If $\pi_1(X, x_0)$ is abelian, show that every covering of X is regular.

Exercise 4 (5 pts):

Remember that an *abelian cover* of X is a G -cover for some abelian group G .

Show that any connected abelian cover of X has the form \hat{X}/H where \hat{X} is the universal abelian cover and $H < H_1X$ is some subgroup of the first homology group of X .