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Topology
WS 10/11

Exercise Sheet 1

Due in tutorials on 27 October 2010

Exercise 1:

On any set X we can define the *cofinite topology* as follows. $C \subset X$ is closed if and only if $C = X$ or C is finite. Prove that this topology is well defined. Is X with this topology Hausdorff?

Exercise 2:

Let $f : X \to Y$ be a continuous map between topological spaces. Prove that if X is compact, then $f(X)$ is compact.

Exercise 3:

Give $\{0,1\}$ the discrete topology. On the product space $X := \mathbb{R} \times \{0,1\}$ we define the following relation:

$$(x,a) \sim (y,b) \iff (x,a) = (y,b) \quad \text{or} \quad x = y < 0$$

Prove that $X/\sim$ is the union of two open sets homeomorphic to $\mathbb{R}$. Is it Hausdorff? (Motivate your answer.)

Exercise 4:

Let $f : X \to Y$ be a map between topological spaces.
We say that f is *open* if for every open set $U \subset X$, $f(U)$ is open in Y.
Similarly, f is *closed* if for every closed set $V \subset X$, $f(V)$ is closed in Y.
For a continuous bijection $f : X \to Y$, prove that the following conditions are equivalent:

1. f is a homeomorphism
2. f is open
3. f is closed