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http://www.math.tu-berlin.de/~sullivan/L/10W/DG2

9. Übung Differentialgeometrie II: Mannigfaltigkeiten

(distance, Poincare half-plane part 1, infinite Moebius strip)

Hausaufgaben

1. Aufgabe (5 Punkte)

Suppose that (M, g) and (N, h) are two Riemannian manifolds and $f : M \mapsto N$ is a diffeomorphism. Prove that f is distance preserving, i.e.

$$d_M(p, q) = d_N(f(p), f(q)),$$

for all $p, q \in M$, if and only if f is an isometry, i.e. $f^*h = g$.

2. Aufgabe (5 Punkte)

We consider the Poincare half-plane $\mathbb{H}^2 := \{(x, y) \in \mathbb{R}^2 : y > 0\}$ with the smooth structure induced from $\mathbb{R}^2$. The Riemannian metric on $\mathbb{H}^2$ is defined by

$$ds^2 = \frac{1}{y^2}(dx^2 + dy^2).$$

1. Consider the curves $\gamma_1 : [0, \alpha] \mapsto \mathbb{H}^2$, $t \to r(\cos t, \sin t)$ and $\gamma_2 : [t_1, t_2] \mapsto \mathbb{H}^2$, $t \to (0, t)$. Show that $L(\gamma_1) = \log\tan(\frac{\alpha}{2} + \frac{\pi}{4})$ and $L(\gamma_2) = \log(\frac{t_2}{t_1})$, where $L(\gamma_i)$ is length of γ_i.
2. Show that $d((0, t_1), (0, t_2)) = \log(\frac{t_2}{t_1})$ for $t_1, t_2 > 0$. Show that the maps $(u_1, u_2) \to \lambda(u_1, u_2)$ with $\lambda > 0$, $(u_1, u_2) \to (u_1 + c, u_2)$ with $c \in \mathbb{R}$, $(u_1, u_2) \to (-u_1, u_2)$ and $(u_1, u_2) \to \frac{1}{u_1^2+u_2^2}(-u_1, u_2)$ are isometries of $\mathbb{H}^2$.

3. Aufgabe (5 Punkte)

Let $C := \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}$ be a right circular cylinder, and let $A : C \mapsto C$ be a symmertry with respect to the origin $0 \in \mathbb{R}^3$, that is, $A(x, y, z) := (-x, -y, -z)$. Let M be the quotient space of C with respect to the equivalence relation $p \sim A(p)$, and let $\pi : C \mapsto M$ be the projection $p \to [p]$.

1. Show that it is possible to give M a differentiable structure such that π is a local diffeomorphism.

2. Prove that M is non-orientable.

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