TECHNISCHE UNIVERSITÄT BERLIN — WS 10

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`http://www.math.tu-berlin.de/~sullivan/L/10W/DG2`

8. Übung Differentialgeometrie II: Mannigfaltigkeiten

(partition of unity, isometries)

Hausaufgaben

1. Aufgabe (5 Punkte)

Let N^k be a compact regular submanifold of M^n and X a smooth vector field on N. Prove that X can be extended to a smooth vector field on M. Does N^k really needs to be compact?

Hint:

For each $p \in N$ choose a neighborhood U_x of x in M and a preferred coordinate chart $\phi_x \colon U_x \to \mathbb{R}^n$. Use a partition of unity subordinate to the covering of M consisting of the sets U_x and the set $M \backslash N$.

2. Aufgabe (5 Punkte)

Prove that for any connected manifold M and any pair of points p and q in M there exists a diffeomorphism f that takes p to q. Give a counterexample for not connected manifolds.

3. Aufgabe (5 Punkte)

Give $\mathbb{S}^n$ the round metric induced by the standard embedding in $\mathbb{R}^{n+1}$. Prove that the antipodal mapping $A : \mathbb{S}^n \mapsto \mathbb{S}^n$ given by $A(p) := -p$ is an isometry of $\mathbb{S}^n$. Use this fact to introduce a Riemannian metric on the real projective space $\mathbb{RP}^n$ such that the natural projection $\pi : \mathbb{S}^n \mapsto \mathbb{RP}^n$ is a local isometry.

Gesamtpunktzahl: 15