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WS 10
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http://www.math.tu-berlin.de/~sullivan/L/10W/DG2

5. Übung Differentialgeometrie II: Mannigfaltigkeiten

(Lie bracket)

Hausaufgaben

1. Aufgabe (5 Punkte)

We consider $X, Y \in \mathfrak{X}(\mathbb{R}^2)$,

$$X = y\frac{\partial}{\partial x}, \qquad Y = \frac{\partial}{\partial y}.$$

Find the flow with infinitesimal generator X and use it to calculate $L_X Y$ from the definition. Compare with $[X, Y]$ calculated directly. Do the same with the roles of X and Y exchanged.

2. Aufgabe (5 Punkte)

Suppose U, ϕ is a coordinate neighborhood on M, $X, Y \in \mathfrak{X}(M)$, and $E_1, \dots, E_n$ the coordinate frames, and note that $[E_i, E_j] = 0$ on U. If $X = \sum_i \alpha^i E_i$ and $Y = \sum_j \beta^j E_j$ on U, then show that

$$[X, Y] = \sum_{i,j} (\alpha^i \frac{\partial}{\partial x^i}\beta^j - \beta^i \frac{\partial}{\partial x^i}\alpha^j) E_j$$

on U.

3. Aufgabe (5 Punkte)

Show that $(L_X Y)_p$ depends on the fact that we use vector fields, that is, if $X, \tilde{X}$ agree at p but are not the same as vector fields, then $(L_X Y)_p$ may differ from $(L_{\tilde{X}} Y)_p$.

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