TECHNISCHE UNIVERSITÄT BERLIN WS 10
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http://www.math.tu-berlin.de/~sullivan/L/10W/DG2

2. Übung Differentialgeometrie II: Mannigfaltigkeiten

(product manifolds, smooth maps, embeddings)

Hausaufgaben

1. Aufgabe (5 Punkte)

Prove the following theorem from class, which defines the product of smooth manifolds.
Let M^m and N^n be smooth manifolds. The topological product $M \times N$ is a topological $(m+n)$-manifold. (You do not have to show this.) Let $(U_i, \varphi_i)_i \in I$ be a smooth atlas for M and $(V_j, \psi_j)_{j\in J}$ a smooth atlas for N. Then

$$(U_i \times V_j, \varphi_i \times \psi_j)_{(i,j)\in I\times J}$$

is a smooth atlas for $M \times N$.
(Here $\varphi_i \times \psi_j \colon U_i \times V_j \to \mathbb{R}^m \times \mathbb{R}^n = \mathbb{R}^{m+n}$ with $(\varphi_i \times \psi_j)(x, x') = (\varphi_i(x), \psi_j(x'))$.)

2. Aufgabe (5 Punkte)

Let M_1, M_2, N be smooth manifolds. For $i \in \{1, 2\}$ we define $p_i \colon M_1 \times M_2 \to M_i$ by $p_i(x_1, x_2) = x_i$. Let $f \colon N \to M_1 \times M_2$ be a map. Prove that f is smooth if and only if $p_1 \circ f$ and $p_2 \circ f$ are smooth.

3. Aufgabe (5 Punkte)

Let X, Y be Hausdorff spaces and $f \colon X \to Y$ a continuous map. Prove that the following statements are equivalent.

1. For every $x \in X$ and every neighbourhood U of x there is a neighbourhood V of $f(x)$ such that $f(X \setminus U) \cap V = \emptyset$.
2. f is a (topological) embedding, that is, a continuous injection which is a homeomorphism onto its image.

Gesamtpunktzahl: 15