TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Prof. Dr. John M. Sullivan
Dott. Matteo Petrera
http://www.math.tu-berlin.de/~sullivan/L/10S/Geo2/

Geometry II
SS 10

Exercise Sheet 5

Exercise 1: Area of convex planar polygons. (6 pts)

Let $p = (p_1, ..., p_n = p_0)$ be a convex planar polygon. Let ϕ_i be the turning angle at p_i and let $\ell_i = |p_{i+1} - p_i|$ be length of the i-th edge.

In lecture, we discussed an analog of the *Heintze-Karcher inequality* for p: in terms of the curvature density κ_i along the i-th edge defined as $[\tan(\phi_i/2) + \tan(\phi_{i+1}/2)]/\ell_i$, we bounded the area enclosed by p as follows: $\mathrm{Area}(p) \leq \sum_{i=0}^{n-1} l_i/\kappa_i$.

Show that we have equality here if and only if p has an inscribed circle (tangent to each of the n edges).

(Hint: the medial axis (or cut locus) of p is always a tree with n "leaf edges" along the angle bisectors at the vertices p_i. Our estimate is sharp if and only if this tree has no internal edges, i.e. just one internal vertex.).

Exercise 2: Gauss curvature of one quarter of the unit ball. (6 pts)

Consider the set $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1, x \geq 0, y \geq 0\} \subset \mathbb{R}^3$, which is one quarter of the unit ball. Find the Gauss curvature of its piecewise smooth boundary.

Exercise 3: Gauss curvature of a polyhedron. (6 pts)

Let v be a vertex of a regular icosahedron. Its five neighbors form a regular pentagon in a plane P. Now reflect v (and the five incident triangles) across P to ge a new (nonconvex) polyhedron (an "icosahedron with a dimple").

1. What is the total Gauss curvature?

2. What is the total absolute Gauss curvature?

Due: Tutorial on 24.06.10