Exercise 1: Distortion of an infinite polygon. (6 pts)

Given $\epsilon > 0$ find an “infinite polygon” $p = (p_1, p_2, \ldots)$ in $\mathbb{R}^2$ with $p_n \to (0, 0)$ such that $TC(p) = \infty$ and $\delta(p) < 1 + \epsilon$.

(Hint: Let the vertices alternate between two nearby rays out of $(0, 0)$).

Exercise 2: Schur’s Theorem. (6 pts)

Let $\gamma \subset \mathbb{R}^d$ be an arc with length $2\alpha$, $\alpha < \pi$. Suppose $\gamma$ has curvature $\kappa \leq 1$. Use Schur’s Theorem to show that the distortion of $\gamma$ is at most $\alpha / \sin \alpha$.

(Note: The condition $\kappa \leq 1$ should be interpreted as saying that the total curvature of any subarc is at most its length. For a $C^2$ curve, this is equivalent to saying that the ordinary curvature $\kappa := |T'|$ is bounded by 1.)

Exercise 3: Distortion of a comet shaped curve. (4 pts)

Let $\gamma : S^1 \to \mathbb{R}^2$ be the closed curve in the figure, consisting of a semicircle $A$ and a polygonal section, consisting of two line segments $P_1$ and $P_2$ with the same length. Let $\phi$ be the exterior angle. Compute the distortion $\delta(\gamma)$.

\[\phi\]

\[A\]

\[P_1\]

\[P_2\]

\[C(c)\]

Due: Tutorial on 27.05.10