TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Geometry II
SS 10

http://www.math.tu-berlin.de/~sullivan/L/10S/Geo2/

Exercise Sheet 3

Exercise 1: Curves of finite total curvature in $\mathbb{R}^3$. (4 pts)

Find an explicit example of a curve of finite total curvature in $\mathbb{R}^3$ whose projection to the xy-plane has infinite total curvature. In particular, find a sequence of points $p_k = (x_k, y_k, z_k)$ approaching $p_0 = (0, 0, 0)$ such that the "infinite polygon" $p_1, p_2, ..., p_0$ has finite total curvature but its projection does not.

Exercise 2: Cauchy-Crofton formula. (4 pts)

1. For $k < d$ prove the following analog of the *Cauchy-Crofton formula*:

   There is some constant c_k^d such that given any curve γ in $\mathbb{R}^d$, its length is c_k^d times the average length of its projections to k-planes.

2. Find c_1^3 and c_2^3.

(Note: for $j < k < d$ we have $c_j^d = c_j^k c_k^d$, by projecting a curve in $\mathbb{R}^d$ first to a k-plane and then to a j-plane.)

Exercise 3: Closed convex curve with constant width. (2 pts)

Let γ be a closed convex curve of constant width d. Prove that the length of γ is πd, just as for a circle.

Due: Tutorial on 11.05.10