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**Geometry I**
WS 09/10

http://www.math.tu-berlin.de/~sullivan/L/09W/Geo1/

# Exercise Sheet 9

**Exercise 1: Correlations.** (4 pts)

A *correlation* is by definiton a point-to-line and line-to-point transformation which dualizes incidences.

Prove that under a correlation, concurrent lines are mapped to collinear points.

**Exercise 2: Dual conics.** (6 pts)

Let $\gamma := \{[x] \in \mathbb{R}P^2 : x^T Ax = 0\} \subset \mathbb{R}P^2$ be a non-degenerate conic. For every $P = [x] \in \gamma$ let $\ell_P := \{[y] \in \mathbb{R}P^2 : x^T Ay = 0\} \subset \mathbb{R}P^2$ denote its tangent line and $P^*$ be the dual of $\ell_P$.

1. Prove that $P^* = [Ax]$;
2. Prove that $\gamma^* := \{P^* : P \in \gamma\} \subset \mathbb{R}P^2$ is a non-degenerate conic (called the *dual conic*);
3. By construction, any point of a conic dualizes to a tangent of the dual conic and viceversa. Dualize Pascal's Theorem (you will end up with Brianchon's Theorem).

**Exercise 3: Conics and quadrilaterals.** (4 pts)

Let $\gamma \subset \mathbb{R}P^2$ be a non-degenerate projective conic through the vertices of a quadrilateral $ABCD$. Let $\ell$ be the line passing through the points $AC \cap BD$ and $AD \cap BC$. Prove that the tangents to $\gamma$ at $A$ and $B$ intersect at a point on $\ell$.

**Exercise 4: Conic through five points .** (4 pts)

Let $A = [1, 0, 0], B = [0, 1, 0], C = [0, 0, 1], D = [1, 1, 1], E = [2, -1, -1]$ be points in $\mathbb{R}P^2$. Find the equation of a conic $\gamma$ (if exists) passing through $A, B, C, D, E$.

Due: Tutorial on 22.01.10