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Geometry I
WS 09/10

Exercise Sheet 7

Exercise 1: Collinearity. (4 pts)

Let $\ell_1, \ell_2, \ell_3, m_1, m_2, m_3$ be distinct lines in a projective plane such that

$$\begin{cases} \ell_1 \cap \ell_2 \cap \ell_3 \neq \emptyset, \\ m_1 \cap m_2 \cap m_3 \neq \emptyset, \\ \ell_1 \cap \ell_2 \cap m_1 = \emptyset. \end{cases}$$

Let $p_{ij} := \ell_i \cap m_j$. Prove that the three points $\overline{p_{13}p_{32}} \cap \overline{p_{23}p_{31}}$, p_{11} and p_{22} are collinear. (Hint: use Pappus Theorem).

Exercise 2: Intersections. (4 pts)

Find the equation of a projective line in $P^3(\mathbb{R})$ which intersects the projective lines

$$\begin{cases} x_1 = 0, \\ x_2 + x_3 = 0, \end{cases} \qquad \begin{cases} x_4 = 0, \\ x_2 - x_3 = 0, \end{cases} \qquad \begin{cases} x_2 - x_1 = 0, \\ x_4 - x_3 = 0. \end{cases}$$

(Note that there are many such lines).

Exercise 3: Projective transformations. (4 pts)

Let $Q := \{[0,1], [1,0], [1,2], [2,1]\} \subset P^1(\mathbb{R})$. Consider the projective transformations $\tau : P^1(\mathbb{R}) \to P^1(\mathbb{R})$ such that $\tau(Q) \subseteq Q$.

1. How many such transformations τ exist?
2. Find explicit formulas for these transformations τ.

Exercise 4: Cross-ratio. (2 pts)

Find all $t \in \mathbb{C}$ such that there exists a projective transformation $\tau : P^1(\mathbb{C}) \to P^1(\mathbb{C})$ with

$$\tau(0) = 0, \qquad \tau(1) = 1, \qquad \tau(t) = 2, \qquad \tau(2) = 6 - t.$$

Exercise 5: Cross-ratio. (2 pts)

Let ℓ, ℓ' be two lines in a projective plane and $p := \ell \cap \ell'$. Let a, b, c and a', b', c' be distinct points on ℓ, resp. on ℓ', all of them different from p. Prove that

$$\overline{aa'} \cap \overline{bb'} \cap \overline{cc'} \neq \emptyset \quad \Leftrightarrow \quad \mathrm{cr}(a,b,c,p) = \mathrm{cr}(a',b',c',p).$$

Due: Tutorial on 11.12.09